Chapter 2

The Vastness of Space

Location and size are important to the habitability of a planet, just like in real estate. I discuss these topics in Chapters 7, 8, and 9. For now I point out the basic issues. Planets that are too close to their star, like Venus, become too hot for life. Those that are too far away may have frozen surfaces. Small objects, like the Moon, cannot hold atmospheres. Large planets, like Jupiter, have massive gas atmospheres with no solid surfaces. The size of stars matters also. Large bright stars as discussed in Chapter 6 burn out and blow up over a brief geological time and are hence inauspicious places to search for life. Stars somewhat smaller than the Sun will last for much larger than the present age of the universe.

Astronomers needed to develop methods starting with the size of the Earth to determine the distances to and find the sizes of the Moon, the Sun, and the planets. After Bruno death, they developed methods to find the distances to the stars. They needed this information to appraise whether the Sun is in fact a star. We need this information to appraise the habitability of objects both within and beyond our solar system.

Astronomers developed many of the techniques in antiquity. They make nice do-it-yourself activities. Other activities arise from work that followed the death of Bruno.

Practical naked-eye astronomy

As discussed in Chapter 1, astronomy began has a practical endeavor for keeping track of time and later for navigation. The sky is there for all who take time to look. It provides a good daily clock and a good calendar for the seasons.

(D2.01) Time of day. We consider a shepherdess who tends sheep in a mid-northern latitude field, say near Boston, Toronto, Tokyo, London, San Francisco, or New York. She has a good view of the horizon and says put during the night. Some stars rise in the east and set in the west (Figure 1). A star that rises at 6 PM due east near sunset is due south at midnight and sets near sunrise at about 6 AM. Another star is low to the horizon and due north at sunset. It moves counterclockwise and is at its farthest point east at midnight. It is due north and high in the sky at sunrise. The pole star, called Polaris, stays due north and fixed to the degree that our shepherdess can casually observe.

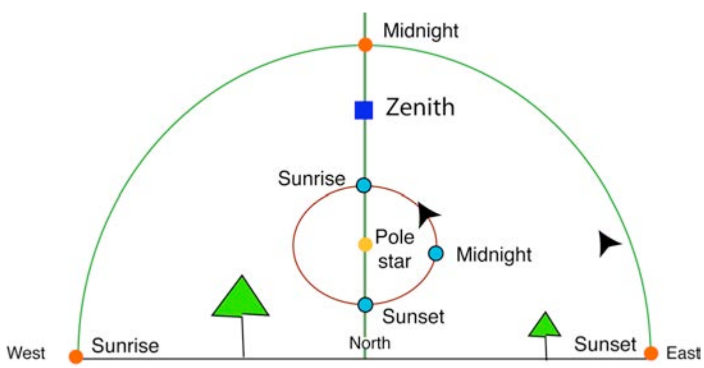


Figure 1: The sky looking north. Stars follow counterclockwise paths. Some rise in the east and set in the west. Others circle the pole star and never set. Astronomers call the vertical point in the sky "the zenith." The red star is south of the zenith at midnight.

The sun rises in the east at about 6 AM. Like the (red) star that rose in the east, it moves gradually higher in the sky. It is high in the sky and due south at noon. During, the afternoon the sun becomes lower in the sky. The sun sets at about 6 PM in the west. Shadows let one observe the direction and height of the sun in the sky without looking directly at it. Long shadows point west at sunrise. Short shadows point north at noon. And long shadows point east at sunset. A sundial accurately monitors this progression.

These events are regular. Our shepherdess need merely remember what happened on the previous nights and days to foretell dawn and sunset. She does need any knowledge of modern astronomy.

Time of year. Seasonal events are also quite regular and obvious at mid-latitudes. We are all familiar with the changing weather and with the changing length of day and night. We all have seen the sun high in the summer sky and low in the winter sky. The changing positions of sunrise and sunset on the horizon are less obvious. I stick here to actual observations. If you are unfamiliar with astronomy, check out the Primer on Seasons at the end of this chapter.

I follow a year from on the start of spring on about March 21. The sun rises due east and sets due west. There are 12 hours each of day and night. During the next three months the sun rises progressive farther north of east and sets progressively farther north of west. It becomes progressively higher in the south at noon on each day. The days become longer and the nights shorter. Frost ceases to form at night and afternoons become uncomfortably hot.

On the start of summer about June 21, the sun reaches its highest point in the sky and rises and sets at its farthest points north of east and west. This is the longest day of the year. Over the next 3 months, days grow shorter and the sun becomes progressively lower in the sky. At the start of fall about September 21, it again rises due east and west like it did 6 months previously. Day and night are again each 12 hours.

During the next 3 months days continue to become shorter and nights longer. The sun rises and sets south of east and west and is low in the sky. At the start of winter about December 21, the sun reaches its lowest place in the sky. It is visible during brief days followed by long nights. Days then become longer returning to the start of the next spring.

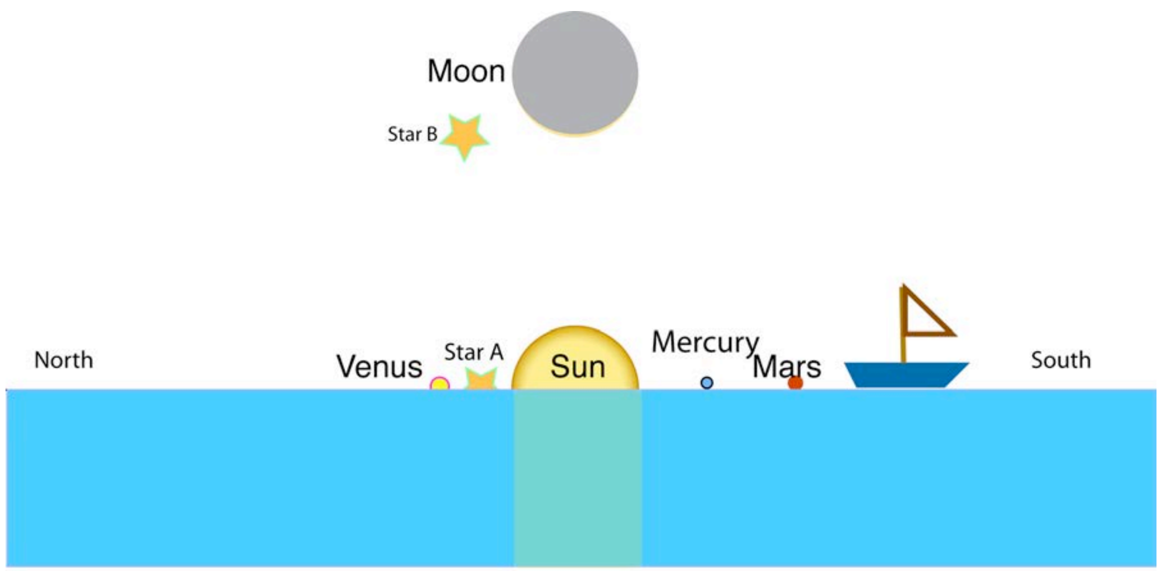


Figure 2: The sky looking east at dawn. The planets Venus, Mercury, and Mars rise with the sun on this day at the same time as a star A. Star B and a very thin crescent moon rose just before the Sun along with a second star B.

The ancients used careful observations of the stars improve their calendars. Now consider an observer at the equator. On March 21, he sees the sun rise in the east (Figure 2). A thin crescent Moon is basely visible in the glare. An eclipse of the sun occurs in the early morning. It is total for about a minute. The planets Venus, Mars, and Mercury that rose with the sun today are briefly visible. The stars of the constellation Pisces surround the sun. This is evidence that the stars and planets exist even when we cannot see them during the day.

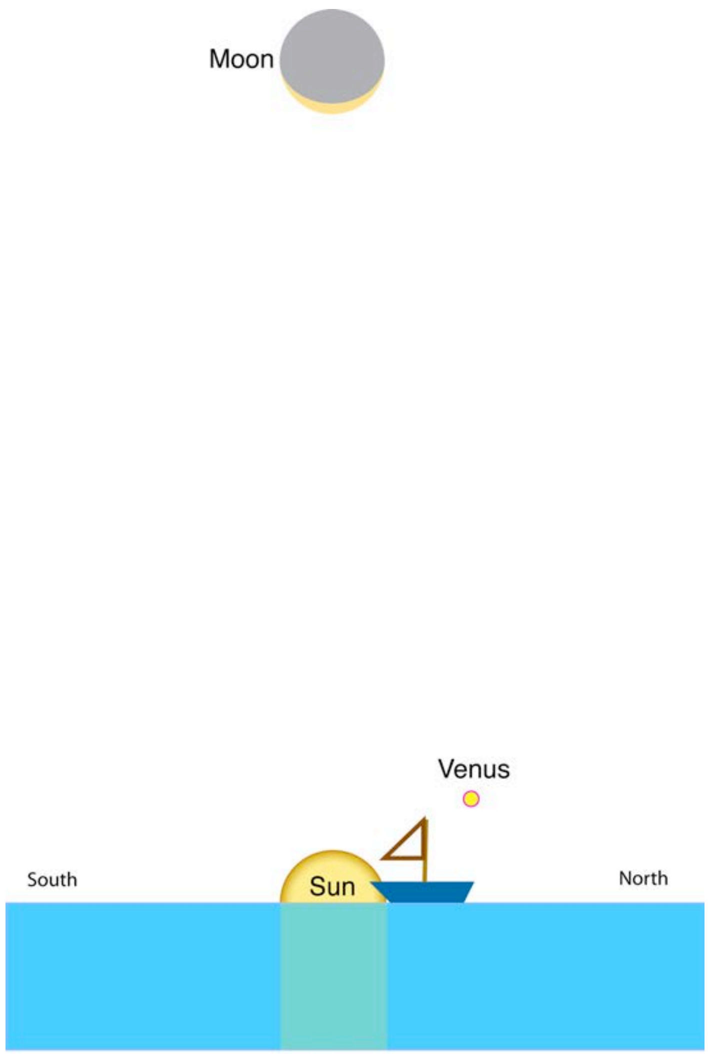


Figure 3: The sky looking west at sunset. The crescent moon will set in less than an hour. Venus sets shortly after the Sun.

That evening, the crescent "new" moon is visible well above the horizon. The planet

Venus is briefly visible if one knows where to look. The next morning the star A that rose with the sun yesterday rises about 4 minutes earlier. Mercury and Mars are barely visible in the glare.

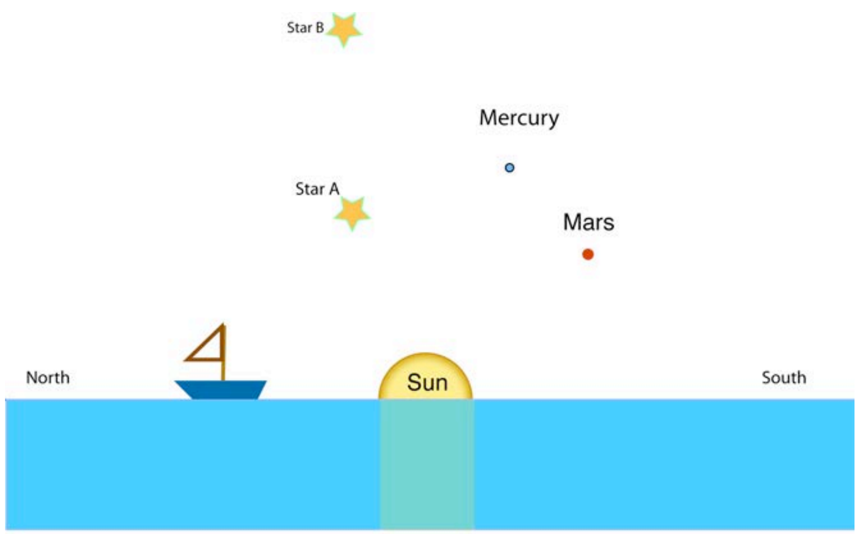


Figure 4: The sky looking east at dawn. The planets Venus, Mercury, and Mars rise with the sun on this day at about the same time as a star.

During the next week the moon rises later each day. It is half full at the end of the week and rises at noon. It is full in another week and rises at sunset. The moon then wanes. It is half full in another week, rising at midnight. The moon wanes into a thin crescent joining the sun in the constellation of Aries at the end of the month. By then Venus is easily visible as an evening "star." Mercury and Mars are less prominent morning "stars."

Modern astronomers retain terms that the ancients used to quantify the position of objects in the sky. I use them here as they facilitate description and so the reader can become familiar with them. The meridian is the imaginary north-south line through the vertical point (zenith) of the sky (the vertical line in Figure 1). Stars pass through their highest point in the sky on the meridian slightly more often than once a day, precisely every 23 hours, 56 minutes, and 04.09074 s. The sun passes the meridian at noon, essentially once every 24-hour day. Stars circle the celestial pole, which is presently near but on exactly at the star Polaris (Figure 1). As discussed later in Chapter 3, the position of the celestial pole changes slowly over time. The ancients learned to recognize the faint stars that then were near the pole.

In practice, ancients determined the position sun against the background of stars by observing the stars visible at dawn and dusk and remembering the positions of the stars lost in the glare. To the degree that one can observe casually over one's lifetime, this path, called the ecliptic, is the same each year. This name indicates that solar eclipses occur when the moon and the sun are aligned on the ecliptic on the new moon. Lunar eclipses occur when the sun and the moon are on the ecliptic on opposite sides of the earth at the full moon. The moon and the 5 naked-eye planets, Mercury, Venus, Mars, Jupiter, and Saturn follow paths near the ecliptic. Astronomers retain the ancient convention of dividing that part of the sky into 12 equal parts of the Zodiac. (Note that I use the Zodiac based on the actual constellations one sees in the sky. The Zodiac signs of astrology columns have the sun enter the sign of Aries at the start of spring; it entered that constellation around the equinox in antiquity.) Eclipses do not occur every month because the moon is not usually right on the ecliptic.

Calendar systems. Ancient societies figured out days and seasons, but faced a variety of problems. There are neither exactly 12 lunar months nor 365 days in a year. One solution, used in the modern Jewish calendar, is to have 12 lunar months per year and insert 13-month years so that the calendar does not get too far out of synch with the seasons. The traditional Chinese calendar has similar rules. The Muslim calendar is 12 lunar months. One must keep track of seasons on one's own. The Mayans had a variety of cycles for describing time, including a 360-day year. They kept accurate track of equinoxes.

It was important to determine the winter solstice in harsh northern climates, probably for morale. Stonehedge in England may have been built for that purpose. Christmas preempted observance of the birth of the sun god Mithra at the winter solstice.

The ancient Egyptians needed to accurately keep track of seasons because their economy depended on the flooding of the Nile and unlike northern Europe their climate is too mild to give obvious precise clues. Attention to having an accurate calendar continued with the Romans. Christians gave particular attention to the timing of the spring equinox before Easter. The Catholic Church introduced the Gregorian calendar during the reformation in 1582 using Copernicus' calculations. It will not need readjustment for a few thousand years. The actual correction is uncertain as the length of the day depends slightly on unpredictable features like the size and position of icecaps on the earth.

Navigation and the size of the Earth. Navigation predates history. Bronze-Age sailors ventured well out of sight of land. Caravan routes extended north-south across Egypt and Arabia. It was essential to get some idea of direction and also of one's position on the Earth. The rising and setting of the Sun provide east and west. Away from the tropics in the northern hemisphere, the highest position of the Sun during the day gives south. Sailors learned to recognize the celestial pole to find north at night.

The sailors knew that the Earth is round and took advantage of the fact. The most obvious evidence is that the hills around a city become visible on the horizon as a ship approaches before the coastline does. For landlubbers, the tip of the mast of an approaching ship becomes visible before the water line (Figure 5).

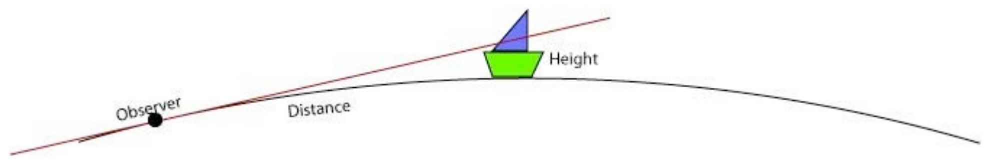
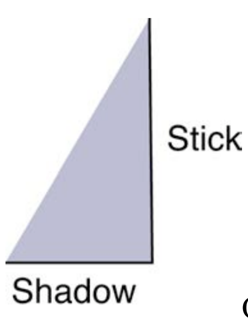


Figure 5: A landlubber views a distant ship that is partly over the horizon. She sees only the top of the sail. With some geometry (See primer at the end of the chapter), she can determine the radius of the Earth knowing the distance to the ship and the height of the seen sail above the water line. How to do this is left as an exercise to the geometrically inclined reader. The mending of light in the Earth's atmosphere as in a mirage makes this method somewhat inaccurate. Not to scale.

Latitude in the northern hemisphere is the angle of the celestial pole above the northern horizon. It increases obviously as one moves northward (Figure 1). More stars cease to rise or to set. Stars in the southern sky disappear below the southern horizon. Going south the progression reverses. Currently the angle of Polaris above the horizon gives a good estimate of latitude. In general, one can obtain a numerical value of latitude from the angle of a star above the horizon when it crosses the meridian using a star chart. With a little geometry and no star chart, one can show that the latitude is average of the two angles above the horizon of a star that does not set (Figure 1).



One can also measure the angle of the sun above the horizon at noon. This is safely done with a stick and shadow. One needs to know the day of the year as the position of the sun in the sky varies. Almanacs provide tables. Going blind from taking sightings on the sun with a sextant was a hazard to navigators in the age of sail.

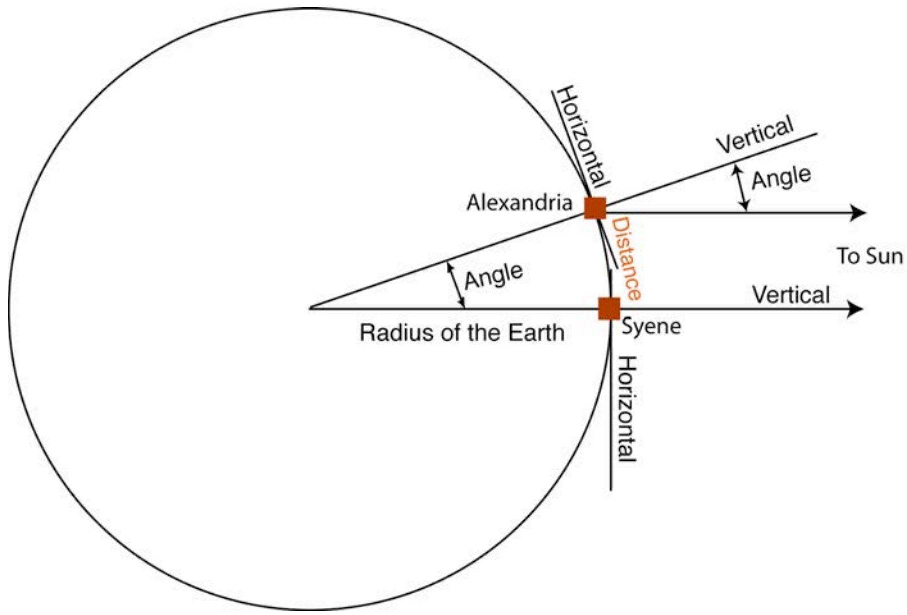


Figure 6: The sun is vertical at Syene and 7° from vertical at Alexandria. As the sun is quite far away compared with the radius of the Earth, the lines pointing to the sun from Alexandria and Syene are essentially parallel. The angle between lines from the center of the Earth to Alexandria and Syene is thus also 7°.

Erastothenes (276-194 B.C.) applied a picturesque version of this method to obtain the circumference of the Earth. He knew that the Sun is vertical in Syene in southern Egypt where its reflection could be seen at the bottom of a deep well, but 7° from vertical at his home in Alexandria at the summer solstice. It was 5000 stadia between these two places. With simple geometry, he computed that the circumference 360° of the Earth is 360/7 times 5000 equal to 260,000 stadia. His estimate was reasonably accurate. We do not know how accurate because historians do not have a precise value of his stadion (or stade) in modern units.

Naked-eye planetary astronomy

The sun borrows of the moon when Diomed keeps his word

Troilus and Cressida, Act V. Scene I

William Shakespeare (1564–1616)
The Oxford Shakespeare, 1914.

Classical and medieval astronomers observed the movements of the Moon and the five naked eye planets, Mercury, Venus, Mars, Jupiter, and Saturn, in addition to the Sun. They interpreted their observations assuming that the Earth is fixed. Shakespeare, as well as his audience, was aware of the basics of naked-eye astronomy. For example, they knew correctly that the Moon shines by reflected light from the Sun.

One must understand the essence and logical basis of ancient and medieval Earth-centered astronomy to understand the arguments that led to its demise. It was based on 4 reasonable and as it turned out testable presumptions:

(1) The Sun, planets, and stars exist even when we cannot see them, as when they are below horizon. This postulate follows the common experience that terrestrial objects are present when they are not visible. Psychologists have shown that infants learn this attribute of common objects in their first year of life.

(2) There is a high degree of regularity of motion even for planets and the moon. With careful observation, one can learn more. Apparent violations of this irregularity, like the appearance of comets, are still frightening to the ignorant. So are infrequent but predicable events like total solar eclipses.

(3) The Moon is many (modern answer 30) Earth-diameters from the Earth. All the other objects are much farther away than the Moon. Astronomers beginning with the Greeks attempted to attach numbers to the distances.

(4) The Earth stays fixed and the objects in the sky move. This was the key sticking point between modern and ancient astronomy. Beginning with the ancient Greeks, some astronomers questioned its validity.

I have already discussed the daily movement of the Sun and stars through the sky. In the geocentric Ptolemaic system, the celestial sphere of the Sun, Moon, planets, and fixed stars rotated en masse around a fixed Earth each day (Figure 7). Modern astronomy retains the celestial sphere as a useful fiction, like the Zodiac, for describing the positions of objects in the sky. As you may have noticed, I have switched to the astronomical convention of capitalizing the Earth, the Moon, and the Sun to indicate that they like the planets are astronomic objects.

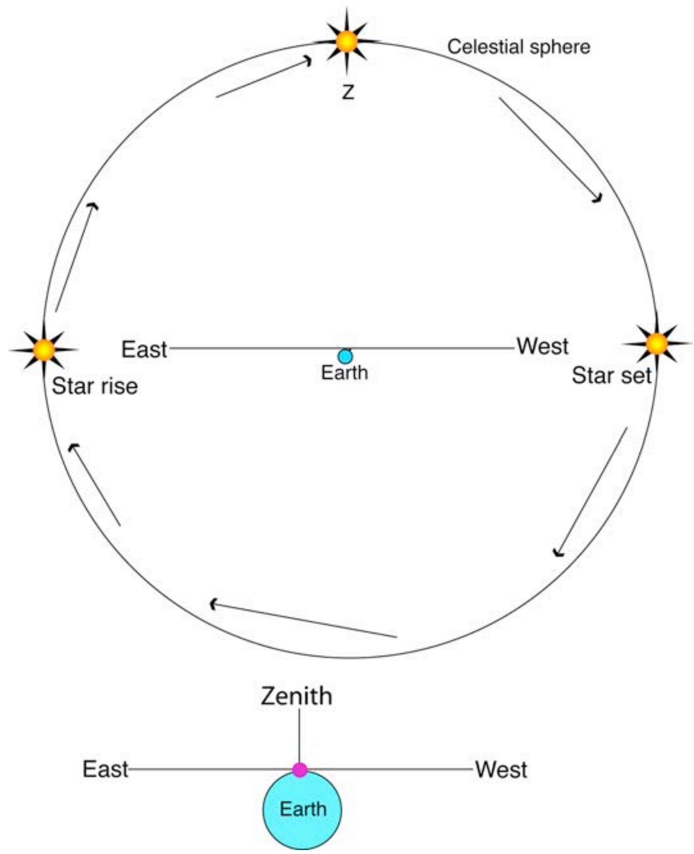


Figure 7: The Ptolemaic system explained the observations available to the Greeks. Stars rise and set viewed from the fixed Earth. The celestial sphere is a real object with the stars attached. The stars rotate clockwise viewed from the North Pole. The horizon, like a leveled tabletop, is parallel to the Earth's surface (below). Stars rise when they pass eastern horizon and set beneath the western horizon. The ancients were aware that the Earth is small enough that one sees half the sky at any one time.

Copernicus, Bruno, and modern astronomy associate the daily motion of the sky with the rotation of the Earth about its axis (Figure 8). The horizon and the meridian rotate with the Earth. The rotation exposes stars above the eastern horizon. The meridian rotates through the star's position. The star sets when further rotation aligns the western horizon with its position.

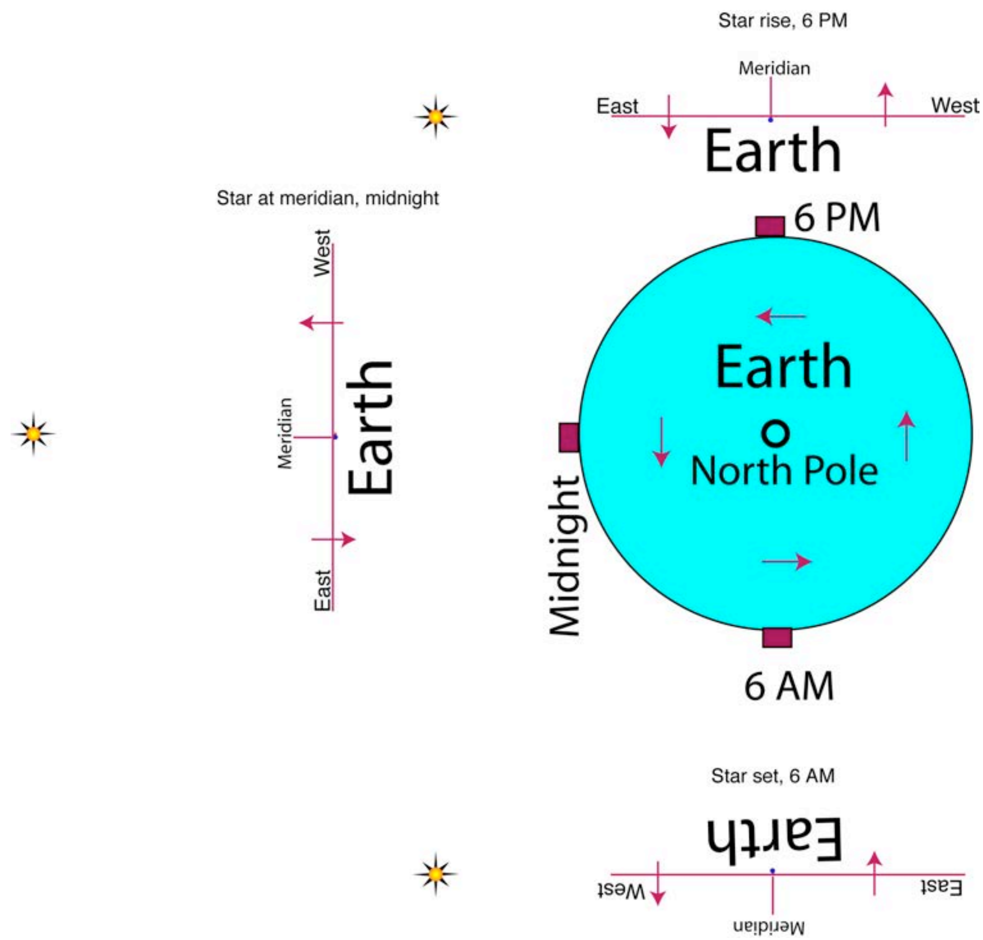


Figure 8: The rotation of the Earth causes stars to rise and set. Viewed from the North Pole the Earth rotates counterclockwise. The horizon like a tabletop rotates with the Earth. A viewer (for simplicity in the drawing at the equator) sees the star rise at 6 PM when the eastern horizon is lined up with the star. Six hours later the star is overhead at the meridian. The star sets at 6 AM when it is lined up with the western horizon. Not to scale. The star is extremely far away compared to the size of the Earth.

The Sun, Moon, and five naked-eye planets known in 1600 move with respect to the fixed stars. The motions and the terminology for dealing with them are somewhat complicated. I begin with modern astronomy that explains this apparent motion with motion of the Earth, the Moon, and the planets.

The Moon orbits the Earth each month and the other planets along the Earth-Moon system orbit the Sun (Figure 9). The Moon moves monthly steadily east with respect to the fixed stars. That is, it rises later and sets later each day. The Sun moves east setting about 4 minutes later than a nearby star over the course of a day. The Mercury and Venus orbit inside the Earth's orbit. Mercury, the symbol of the messenger of the Greek gods, darts back in forth from being a morning star and to an evening star. It never gets far from the Sun and is not easy to see in the glare of twilight. Venus also alternates between being an evening and a morning star. It moves more slowly and gets far enough from the Sun to be the brightest object other than the Moon in the night sky. It casts a visible shadow in secluded locations on moonless nights. It is visible in daylight if one knows where to look.

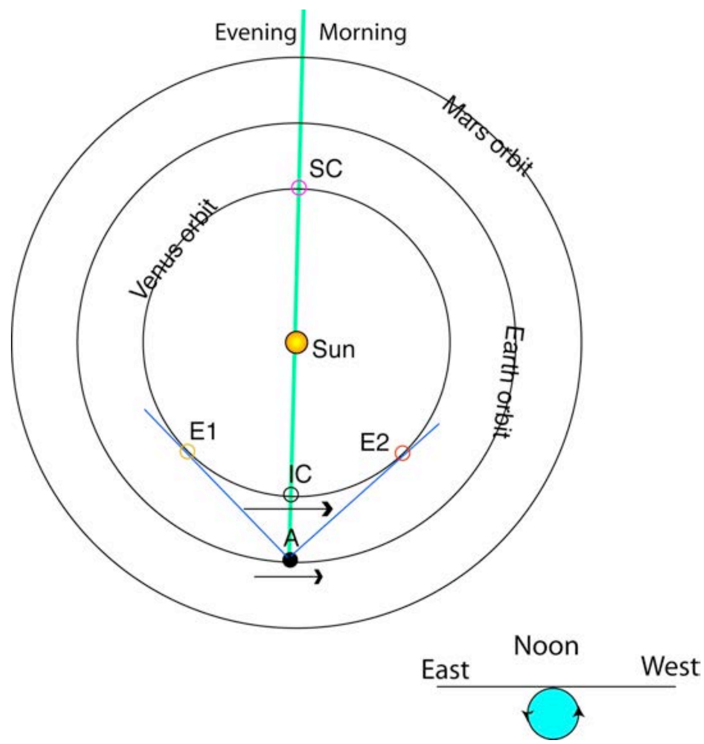


Figure 9: The planets rotate about the Sun counterclockwise viewed from the North Pole in the Copernican system. The diagram is simple showing only Venus, Earth, and Mars. Venus revolves around the Sun faster than the Earth. If the Earth is a point A in its orbit, special positions in Venus' orbit are: IC, inferior conjunction between the Earth and the Sun; SC, superior conjunction, being on the opposite side of the Sun; and E1 and E2, maximum elongation, having the maximum angle from the Sun. Venus overtakes the Earth so that it goes through elongation 1, inferior conjugation, elongation 2 and superior conjunction. The viewpoint of an observer on the Earth at noon is convenient of visualizing the motion of Venus in the sky. Objects east of the Sun set after it and are visible in the evening sky. Objects west of the Sun, rise before it and are visible in the morning sky. At inferior conjunction, Venus moves from being an evening to a morning "star." It moves from a morning to an evening "star" at superior conjunction. Orbits are only mathematical entities, not physical tracks. A spacecraft crossing the orbit of Venus sees nothing special.

Mars, Jupiter, and Saturn orbit outside of the Earth. Their orbital periods (their years) are longer than the Earth year (about 365 and 1/4 days). The outer planets generally move to the east in the sky. Each time that the Earth overtakes one of them (called opposition), the planet moves to the west in what is called retrograde motion.

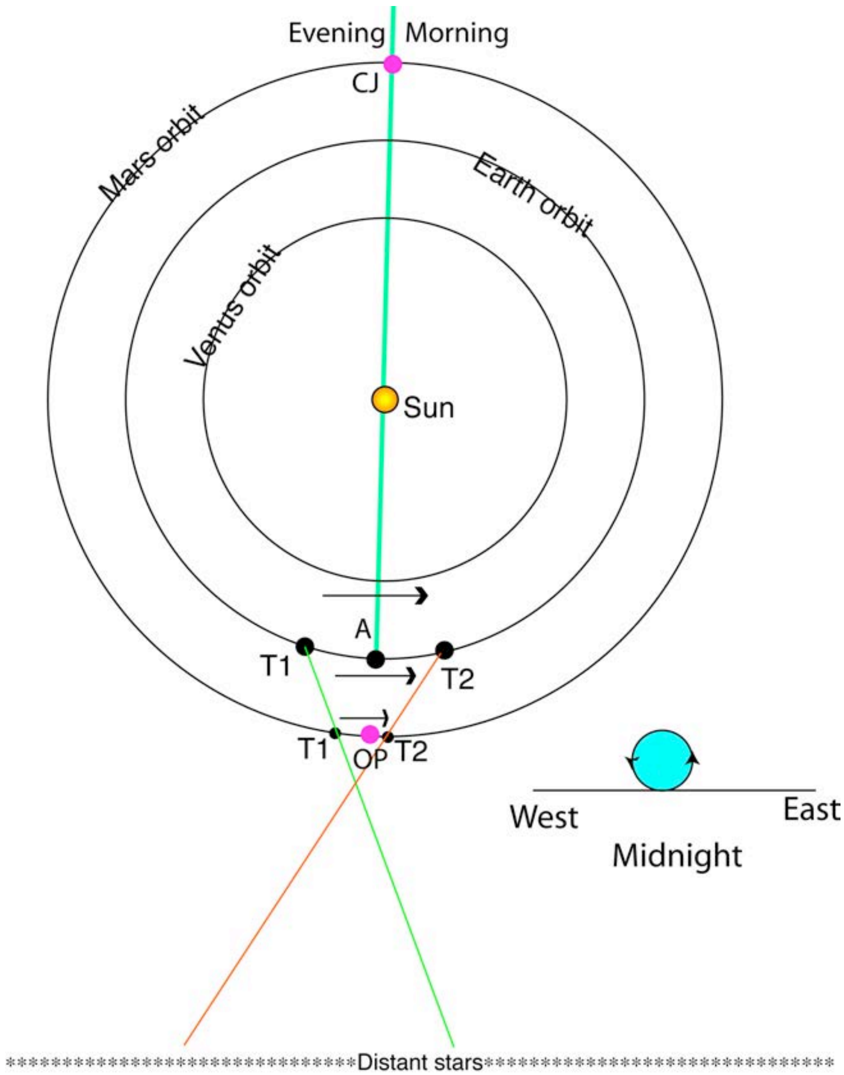


Figure 10: Mars at conjunction (CJ) is on opposite side of the Sun from the Earth. As the Earth overtakes Mars, Mars moves from being an evening "star" to a morning "star" at this time. Mars is on the same side of the Sun at opposition (OP). It crosses the meridian at midnight. Retrograde motion occurs at this time. The Earth overtakes Mars (between times T1 and T2) causing Mars' apparent motion to the west against be background of distant stars. Note that the disk size of both Venus and Mars varies as they move nearer and further away from the Earth.

The Ptolemaic system was adequate for describing the motion of planets in the sky, but became quite contrived when it represented retrograde motion (Figure 11). In addition to revolving on its orbit around the Earth, each outer planet revolved on an epicycle that took it backwards each opposition. Medieval opinion differed on whether the orbits and the epicycles were physical "crystalline" objects or just convenient mathematical entities.

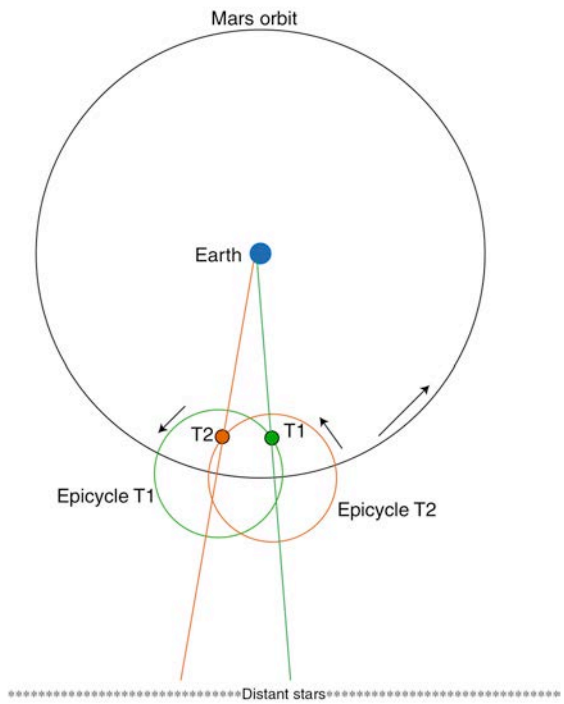


Figure 11: Mars rotates about the Earth counterclockwise viewed from the North Pole in the Ptolemaic system. Retrograde motion occurs because Mars is on an epicycle that rotates clockwise. Some carnival rides have seats that rotate in this way. Many medieval astronomers considered the orbit and the epicycles to be real objects. In practice, a single epicycle did not adequately represent motion; the Ptolemaic system needed more epicycles and a dustbin of other contrivances.

Distance and Parallax. The relative and absolute distances of objects in the sky were a key issue for the debate between Ptolemaics and Copernicans. The Moon is obviously nearer to the Earth than the Sun, which it sometimes eclipses. It also passes in front of the planets and the fixed stars, called occulting them. It was well known to

Aristotle (384-322 BC) that this indicated that the Moon was nearer to the Earth than the stars and the planets. Transits of planets Mercury and Venus in front of the disk of the Sun are visible with the naked eye protected by smoked glass or safely with a pinhole camera, but only if one knows when to look. Occultations of planets by the Sun are not visible to the naked eye. Note that astronomers reserve the term eclipse for when the disk angular diameters are comparable and use transit when the small disk passes in front of the large one and occultation when the small disk passes behind the large one. They also use eclipse when a small object moves into the shadow of a larger one, as in a lunar eclipse.

The ancients used guesswork to assign the slowly moving planets Mars, Jupiter, and Saturn to progressively further distances from the Earth outside the orbit of the Sun. They differed on whether to put Mercury and Venus inside or outside the Sun or to have them circle the Sun.

The ancients had good estimates of the size of the Earth as noted above. They used this as a yardstick to obtain the size and distance of the Moon as we see in this section. They attempted to obtain the distances to the Sun and to the stars.

Classical methods cleverly applied geometry [see Primer on Geometry] taking advantage of parallax (Figure 12). If you are not already familiar with the phenomenon, hold a finger in front of your eyes. Blink one eye and then the other. Your finger will appear to jump back and forth on the background. Now repeat the experiment with distant objects. A distant foreground object will jump slightly on the more distance background, but your eyes cannot resolve the movement. You need a larger baseline. Parallax may become evident if you walk back and forth.

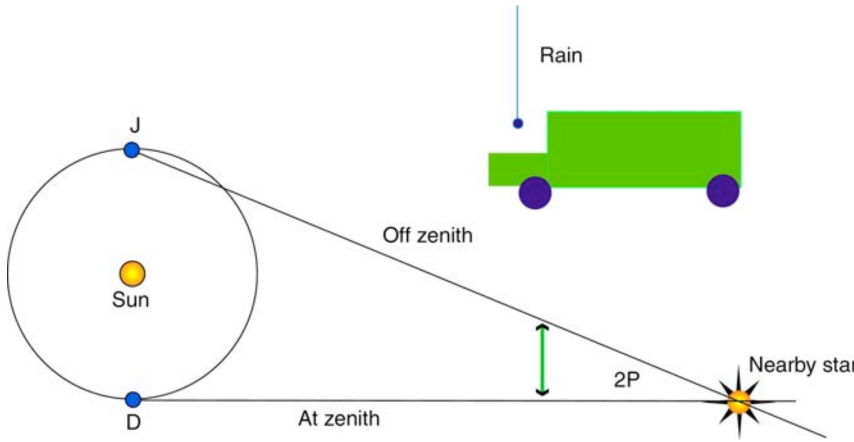


Figure 12: Astronomers determine the distance to nearby stars by measuring the parallax angle against the background of distant stars. The baseline here is 2 AU the distance between the December and June positions of the Earth. They record by convention the half angle P with a baseline of 1 AU. The actual angles are tiny and the stars are moving with respect to the Sun. It takes several years to get a reliable answer. Alternatively they could measure the change of angle relative to the Earth's surface. A star that passes directly overhead at the zenith in December will be slightly off the zenith in June. This method fails because of the aberration of light. Starlight appears to come (slightly) from the direction the Earth is traveling, like rain striking the windshield of a moving truck.

The Earth forms a useful baseline of thousands of kilometers. One may observe objects simultaneously at different points on the Earth, geographic parallax. Accurate timing to do this was impossible in 1600. Today a pair of good amateur telescopes equipped with digital cameras suffices to get a reasonable estimate. Alternatively, one can observe the relative motion of a nearby object like the Moon over a night with respect to the more distant stars (daily parallax). The ancients used a mixture of these methods involving the eclipses of the Sun and the Moon.

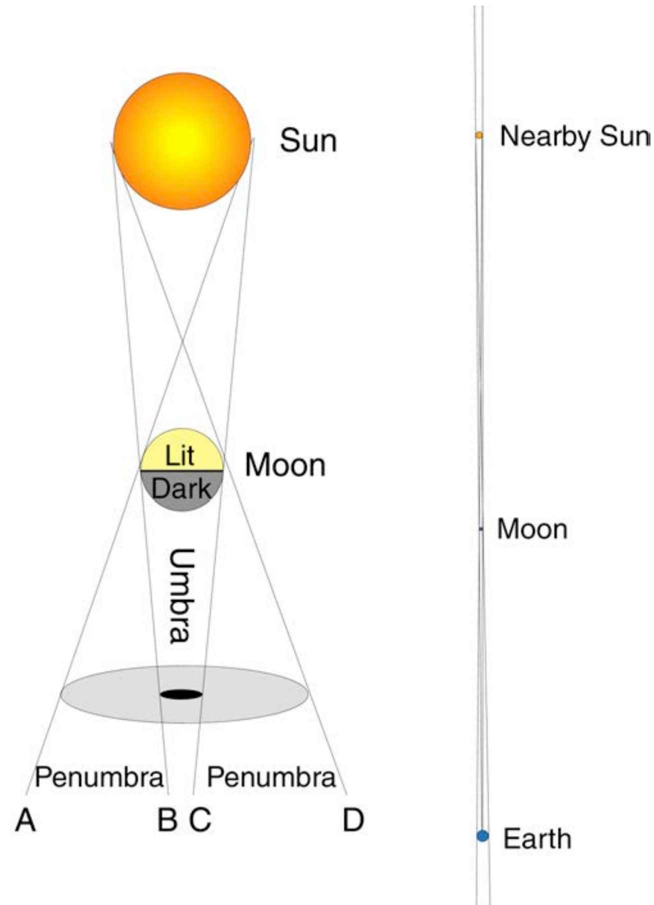


Figure 13: The Moon casts an inner shadow of total eclipse, the Umbra and an outer shadow of partial eclipse the Penumbra (left). The actual disks of the Sun and Moon are about a half degree. The Moon and the Earth are at the true relative scale and distance (right). At totality the umbra is a tiny spot on the Earth. The Sun must be somewhere in the cone formed by the umbra and the disk of the Moon. If the Sun is placed too close to the Earth in the diagram, the entire Earth ends up in the Penumbra, which does not happen. This is evidence that the Sun is in fact very far away. The line A and C are the essentially parallel as are the lines B and D.

Eclipses of the Sun provide methods for estimating the distance to the Moon. I describe a modified and simplified version of the method in antiquity that requires continent-scale communications but not good clocks. The disk of the Moon is a half degree of arc across, usually just big enough to totally eclipse the Sun, also about a half degree (Figure 13). The Moon casts an inner shadow, called the umbra where it totally covers (eclipses) the disk of the Sun and an outer shadow, the penumbra where part of the

Sun's disk is visible (called a partial eclipse). You can see the effect yourself by projecting a flashlight on an object narrower than its beam or by totally or partly covering a distant object with your outstretched hand and closing one eye.

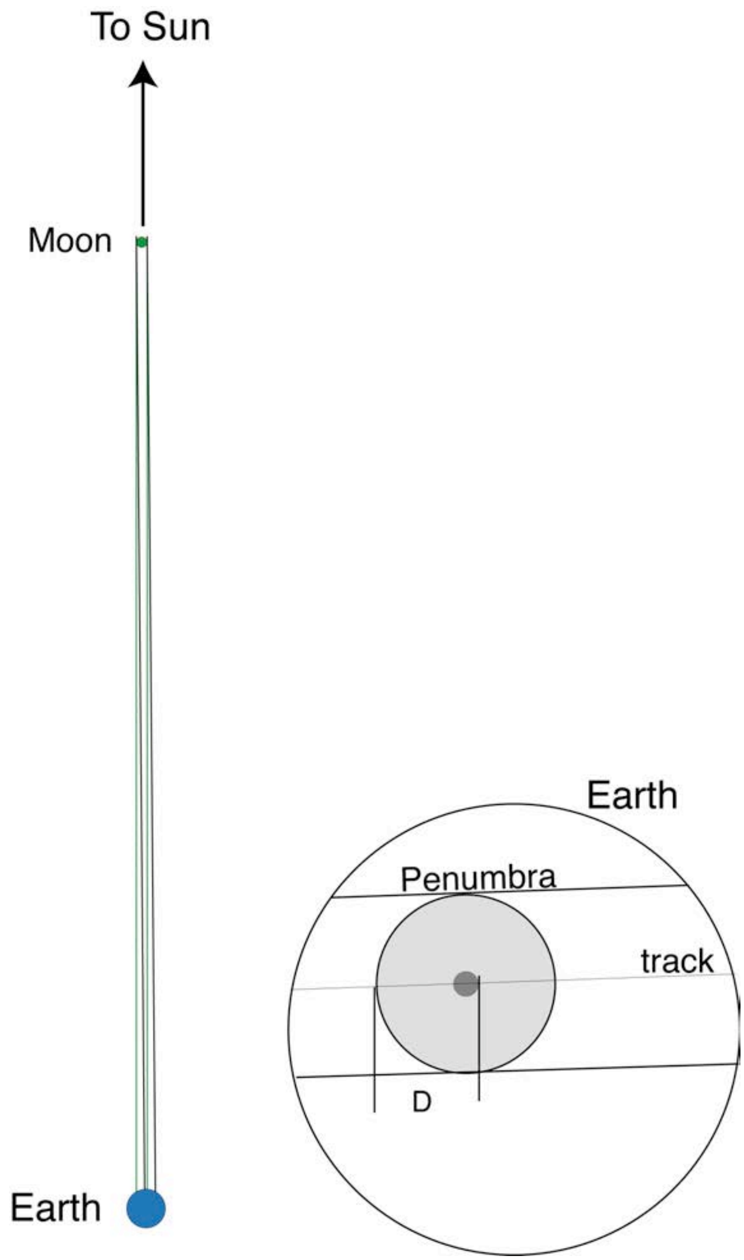


Figure 14: The eclipse of the Sun with the Earth and Moon and their separation are drawn to scale. The Sun is very far away. The diameter of the Moon is the distance D between the edge of the Penumbra and the opposite edge umbra as projected on the disk of the Earth. It was hard to precisely determine this distance in antiquity but they got about the right result using the duration of the eclipse.

A total or near total solar eclipse cannot be missed nor quickly forgotten. In fact, it panics the ignorant. Totality is brief and the band of totality narrow. This information was quickly gathered in antiquity. The band of the partial ellipse is relatively narrow (Figure 14). As the Sun is actually much further away than the Moon, the Moon's radius is given by the projection of a line from the edge of its umbra to the opposite edge of the penumbra on the disk of the Earth (Figure 14). The ancients could gather some information on the size of the area of partial ellipse, but it was inadequate. They relied on a more complex method involving the duration of the eclipse. In contrast, a partial eclipse would cover the whole side of the Earth if the Sun were nearby (Figure 13).

The Earth's umbra completely covers the Moon during a total lunar eclipse indicating that the Earth is the larger body (Figure 15). (An astronaut on the Moon then would see a total eclipse of the Sun by the Earth.) The time for the Moon to pass through the umbra and the penumbra and the curvature of the umbra and the penumbra give the relative sizes of the Moon and the Earth. The ratio of the umbra diameter to the penumbra diameter constrains the relative distances of the Sun and the Moon. In practice, all the ancients could tell is that the Sun is very far away. The line between the edges of the umbra and the penumbra then gives the Earth's diameter.

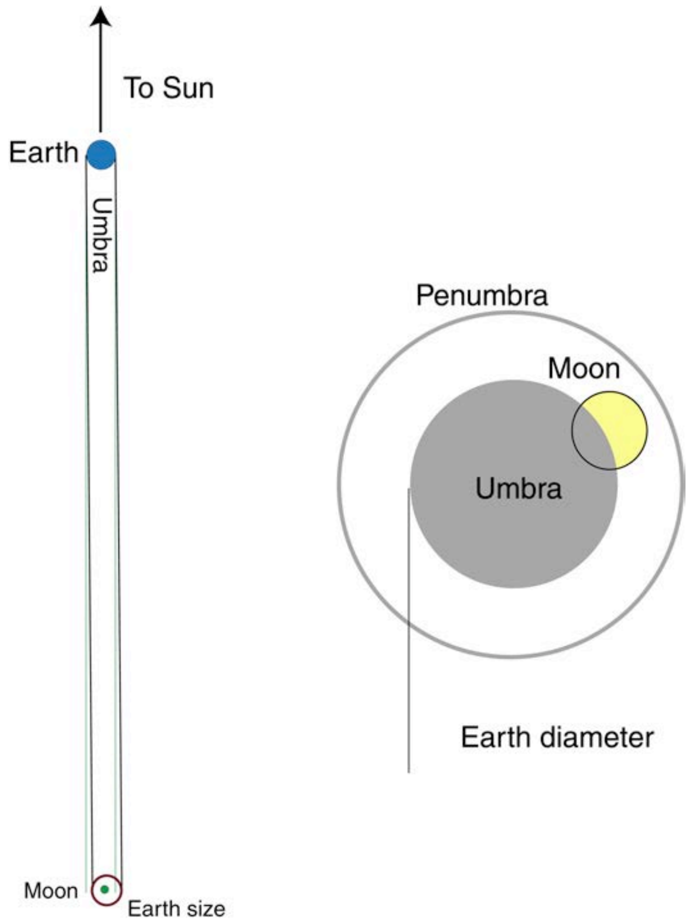


Figure 15: The eclipse of the Moon with the Earth and Moon and their separation distance drawn to scale. The Sun is very far away. The diameter of the Earth is the distance D between the edge of the penumbra and the opposite edge of the umbra. It was easier to use the curvature of the umbra on the Moon in antiquity as it is at present. With some geometry, the Earth's diameter is the umbra diameter plus the Moon's diameter because the disk diameter of the Sun and the Moon are both half a degree. Such methods give the diameter of the Moon as a fraction of the diameter of the Earth. The ancients got reasonable answers. Note: dime (or 2 Euro piece) held at about arms length has about the curvature of the umbra for most people. You can then estimate the number of Moon disk diameters that fit within the dime.

Once one has the diameter of the Moon as a fraction of the Earth's diameter, it is a simple matter to use the fact that the disk diameter of the Moon is a half-degree to get the Earth-Moon distance. Greek estimates were between 20 and 30 (the modern value) Earth diameters. They knew that the distant Sun is much bigger than the Earth.

The size and distance of stars. The Earth is too small to make a useful naked-eye baseline for objects further away than the Moon. Greek and medieval astronomers realized that the Sun is many Earth diameters away and that the stars should show parallax if the Earth in fact orbited the Sun (Figure 12). This parallax would occur even if all the stars were the same distance away on a crystalline sphere. Then a star that was vertical (at the zenith) when it passed meridian in June would no longer be vertical in December. No annual parallax was ever detected.

Bruno and Galileo were well aware of this observation but they (like the Greek astronomer Aristarchus) contended that the stars were so far away that one couldn't see parallax. They had trouble making a convincing argument beyond that their system was simpler and that it seemed silly for the larger luminous Sun to circle the smaller Earth.

From the appearance of the size of the luminous body we cannot infer its true size nor its distance, for just as the case is not the same with an opaque as with a luminous body, so the case is not the same for a luminous, a less luminous, and a very luminous body as to let us estimate the distance or size. Giordano Bruno. *Ash Wednesday Supper*, 1984.

Bruno was well aware of the difference between the absolute brightness (luminosity or light output) of an object and the apparent brightness that one sees at its current distance. He was aware that a given luminosity could be achieved by a small object with great brightness per surface area or by a large object with modest brightness per surface area. He did not mathematically quantify these concepts.

The argument that the Sun is a star is strengthened if when one shows that both have similar luminosity and diameter. This requires measuring the distance to the star, which cannot be done with the naked eye.

However from Bruno's hypothesis, one can make the trial assumption that the Sun is a typical star. That is, the other stars have the same luminosity and diameter as the Sun. Then one can attempt to measure the ratio of apparent luminosity of the Sun to the star or the ratio of their disk diameters. In the first case, one can use geometry to figure out how far the Sun would have to be away to as bright as the star. Disk diameter is straightforward. For example, the disk diameter of the Sun is 0.5°. A star one million times farther away, for example, would have a disk diameter one millionth of 0.5°.

With a little geometry, the ratio of Sun to star distance gives the parallax angle. In practice, the assumption that is Sun is the same as all other stars is not correct. Some of the brightest stars in the sky are much more luminous and much wider than the Sun. There are also many dimmer and smaller stars, some nearby, but no one in 1600 would have started by looking at them. It turns out the nearest star system, Alpha Centauri, contains two sunlike stars and one small faint star. As we see below, one does get tolerable estimates of star distance by assuming the Sun is a typical star, all with the naked eye.

The telescope: A new view of the sky

These experiments on star brightness and disk diameter did not get done immediately after 1600. The invention of the telescope distracted astronomers. Worse yet, the ruckus belatedly awakened the Church to the dangers posed by the Copernican system to its cozy worldview.

Galileo's discoveries. Events in Venice during May 1609 began modern astronomy. A university professor heard a reliable account of a new device that lets one view distant objects. Spyglasses were already on sale in Paris, but Galileo (1564-1642) was impatient. He bought spectacle lenses from a local optician on his return to his home in Padua and made some for himself. The three-power spyglasses were inadequate for viewing distant objects. By trial and error, he determined the relationship of the magnification to the focal lengths of the lenses. The opticians were a hidebound medieval guild, unable to supply stronger lenses. Galileo figured out how to grind better lenses on his own.

His first success was an 8-power instrument, comparable to modern binoculars. He acted quickly before they were commonplace. Always a semi-successful self-promoter and always short of cash, he presented his instrument to the Doge of Venice. He asked that his salary be increased. This happened, but he was told that he would get no further raises.

By November, he had a 20-power instrument. Fatefully he pointed it at the sky. He saw shadows cast by relief on the surface of the Moon and imaged craters. He saw that vast numbers of dim stars became visible as Bruno had expected. Then he pointed the instrument at Jupiter. He resolved that it is a circular disk and hence a spherical body. A line of 3 moderately bright "stars" appeared near the planet. He recorded them, but did not give them much thought. The next night the stars were on the other side of the planet. This was not expected since Jupiter's motion was retrograde. His interest was piqued. Over several nights he found that there are in fact four objects that circle Jupiter. That is, he had found a separate center of rotation independent of the Sun and the Earth. His new objects are the four large moons of the planet Jupiter.

Speed was of the essence. Others would soon have nice telescopes. Galileo rushed a book, *Sidereus Nuncius* (*The Starry Message or Messenger*) with such haste that he did not get the (then well-known) distance from the Earth to the Moon right. Simultaneously obtained a new patron by naming his new moons after the powerful Medici family in Florence and a no-duties professorial position to boot. The Medici made him an international hero. They sent good telescopes and his book to the patrons of science, the crowned heads of Europe. This was probably the first government "outreach" program to disseminate scientific information.

Galileo was quick to voice support for the Copernican system. If Jupiter and its four moons orbit the Sun, so can the Earth and its moon. Other discoveries quickly followed. Galileo found that Venus has phases like the Moon, which is expected from the Copernican system. He found that the disk size of Mars varies as expected (Figure 10). All was there for all to see. (You can easily see numerous stars with good binoculars and the moons of Jupiter if your hands are steady. A small telescope nicely brings in the phases of Venus and shows the planets as disks.)

Kepler's reply to Galileo: Trouble from a safe place. Important news traveled fast in the late Renaissance. In 1610, the Imperial court astronomer in Prague was troubled by news from Italy. An Italian astronomer had found four new planets. If they orbited another star, Bruno was right. The Sun would be but one of an infinity of stars. His friend Johannes Wackher forcefully pointed this out.

Finally the news arrived in the form of Galileo's book sent by the Medici's to the Imperial court. Johannes Kepler was relieved and sent back a generally positive critique to Galileo. He was not bothered by an infinity of stars, but he was sure that the Sun was not one of them. He set out to disprove Bruno's "dreadful" theory once and for all. His reasoning was simple: the disk diameter of stars is about one minute of arc, a 30th of the disk of the Sun. This had been known (incorrectly) from antiquity. It was obvious (correctly) that the Sun is much brighter than 900 (30 squared) bright stars. The simplest way to see this is with a pinhole (Figure 16). The sunlight is dimmed by a factor of 900 when the diameter of the spot is 30 times the diameter of the pinhole. The spot is far brighter than starlight. Like modern science, Kepler's argument stemmed directly from the observations.

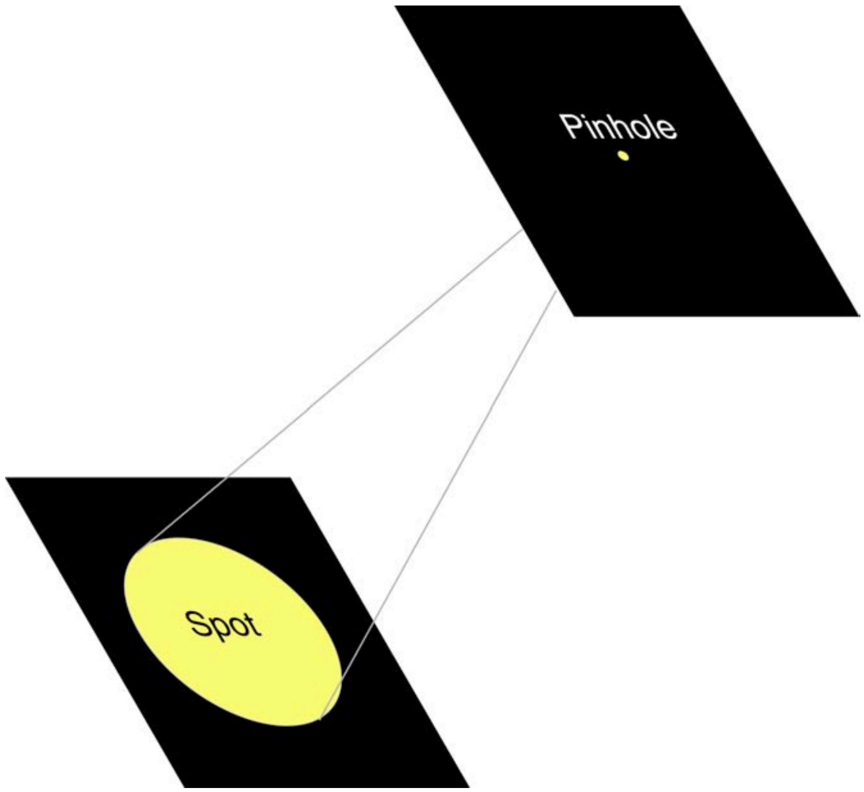


Figure 16: A pinhole camera produces a spot of dimmed sunlight. The spot is as bright as sunlight would be viewed a distance in AU equal to the diameter of the spot divided by the diameter of the pinhole. The pinhole camera produces an image of the Sun. It is a safe way to view a solar eclipse. See the Activities at the end of the chapter for other uses of this versatile instrument.

Besieged by questions from all quarters, Kepler published his letter as a book, *Conversation with the Starry Messenger* to get some relief. The cat was out of the bag. Soon everyone with even a casual interest in the sky was aware of Bruno's infinity of worlds. Kepler's argument turned out to be weak because Galileo was already showing that the apparent disk diameter of stars is far less than one minute of arc. It narrowed with each improvement in the telescope. Only the planets show resolvable disks. Modern equipment barely resolves the disks of some nearby and some every large stars.

Rather than killing Bruno's hypothesis, Kepler's faint damnation linked it forever with the Copernican system. There was no way the Church could suppress the telescope.

They did not even think of trying. It was too useful for military and navigational purposes. It was a fine toy for the trendy. There was no way to keep it pointed away from the sky. The moons of Jupiter had practical applications. Galileo computed accurate timetables. The occultations, eclipses, and transits of the moons occur at essentially the same time everywhere on the Earth. They were a potentially useful clock for finding longitude (See Do It Yourself Box on Navigation), an important task in the age of exploration.

The more conservative Churchmen reacted carefully. Their cozy worldview rested on the fixed Earth. They defended at that point, not at the somewhat roomier Sun-centered universe of Copernicus and Kepler. Explicitly condemning an infinity of worlds or that the Sun is a star would have served only to make even more people aware of these simply expressed ideas. (This is a little like the folly of publishing an exquisitely detailed antipornography law.) Galileo never advocated the former nor even mentioned the latter. He had never cited Bruno. He was in fact a devout practicing Catholic.

The conservative Churchmen set a trap to move the game to their home court with home referees. They condemned the Copernican system on biblical grounds, like God could not have stopped the Sun for Joshua if the Earth moved. These untestable assertions were better suited to the Churchmen than ignorantly going after Galileo's physics and astronomy. (Galileo also had numerous "safe" opponents, who objected on basically scientific grounds.) Galileo did not let their attacks pass. He responded in widely circulated private letters that the religious objections were vestiges from a simpler time and that the Bible should not be taken literally, but he stuck to generalities. He feared that the Church was about put itself in the compromised position of imposing doctrines of faith that would be eventually (as it turned out within that century and conclusively by 1725) disproven by physical measurements. His friend, the Carmelite priest Paolo Antonio Foscarini (ca. 1565-1616), published a detailed account of why Copernicanism did not conflict with the Scriptures. (Like Kepler, he contended that the Sun was much more luminous than a star.) Galileo and Foscarini were treading with their full weights on dangerous ground. Bruno had unsuccessfully raised this dichotomy between religion and philosophy as a defense, but Galileo was confident that he could convince authorities in Rome.

Galileo and the Church after 1615: Forever amber. The Cardinals and the Pope treated Galileo politely in 1616. He was an international hero. As a layman, he enjoyed more leeway than Bruno and Foscarini. Renaissance church dogma was intricate. It was easy to fall into heresy. As a practical matter, the faithful were allowed to recant and repent. In Italy, the dungeon, the stake, and the rack were reserved for the recalcitrant, like Bruno. The Church forbid Galileo from holding or defending Copernican doctrines, but did not specifically mention him in their public edict. It was still OK to use the Copernican system as a useful fiction in calculations. Whether they explicitly banned him from teaching the Copernican system became an issue at his trial. Galileo was free to continue with astronomy as long as he did not advocate that the Earth circles the Sun.

But it was too late for the Church to drown Copernicanism. By 1616, the populace was familiar enough with it that Foscarini needed only a cursory description in his book. Like ordering back the tide, the Church banned Copernicus' book until it was fixed. This had little direct effect on the street because only the most mathematical could understand it anyway. The Church allowed it to be printed with a few changes four years later. (They totally banned it in 1664.) They banned Foscarini's book outright and he died soon after. This temporarily silenced Italian proponents of Copernicanism.

By 1629, Galileo had managed to outflank the restrictions imposed by the Church. He spent endless sessions with the censors preparing his *Dialogue Concerning the Chief World Systems*. He cleverly selected the names of his characters. Salviati and Sagredo were safely dead colleagues of Galileo. Simplicio (490-560, Simplicius in Latin) was an Aristotlean philosopher, safely well known to the censors. Simple can mean compact, elegant, and lucid, like with Newton's law of gravity. But it can mean stupidly simpleminded. Years later this view of Aristotleans became universal; our word dunce comes from late Renaissance belittling of the followers of the Aristotlean John Duns Scotus (ca. 1265-1308).

Galileo could not advocate the Copernican system, but there was nothing to prevent him from making his advocate of the Ptolemaic system, Simplicio, from living up to the second meaning of his name. Simplicio's more inane arguments, like that the Moon is translucent, highlighted his slavish devotion to Aristotelian philosophy. They related only indirectly on the larger issue of whether or not the Earth moves.

Galileo was far more knowledgeable than his censors and was a master at incomplete arguments. He left out the punch line at a point that fooled the censors but let the curious fill in the blanks. He was fortunate that his opponents had already used the diameter of the Sun and the distance from the Earth to the Sun as yardsticks with which to speculate about the size of stars. This let him mention that the disk size of the Sun would be very small if viewed from very far away.

Galileo's arguments about the disk size of stars were a tacit reply to the reasoning in Kepler's *Conversation* that the Sun is not a star. This is as close as Galileo ever got to publicly advocating that the Sun is a star and a valid argument that the stars do not shine from reflected light from our Sun. He noted that the by-then-common telescopes show the planets as disks, but did not resolve the stars as disks.

He noted that the brightness per disk area of Venus, which is near the Sun, is much greater than the brightness per area of Jupiter, which is distant from the Sun. I add a little bit of geometry to help here. The Sun can be considered to illuminate an imaginary sphere at some distance R from the Sun of an object (Figure 17). Remembering geometry, the area of our sphere is 4 times pi times the radius squared or $4\pi R^2$ in mathematically notation. The light per area on the sphere is thus proportional to the inverse of that radius squared.

The reflected light from the object illuminates another imaginary hemisphere (containing the Earth) about radius R (if it is much more distant than the Earth is from the Sun). The light per area on this hemisphere is also proportional to the inverse of that radius squared. The net effect is that the reflected light from a distant object decreases with the inverse distance to the fourth power.

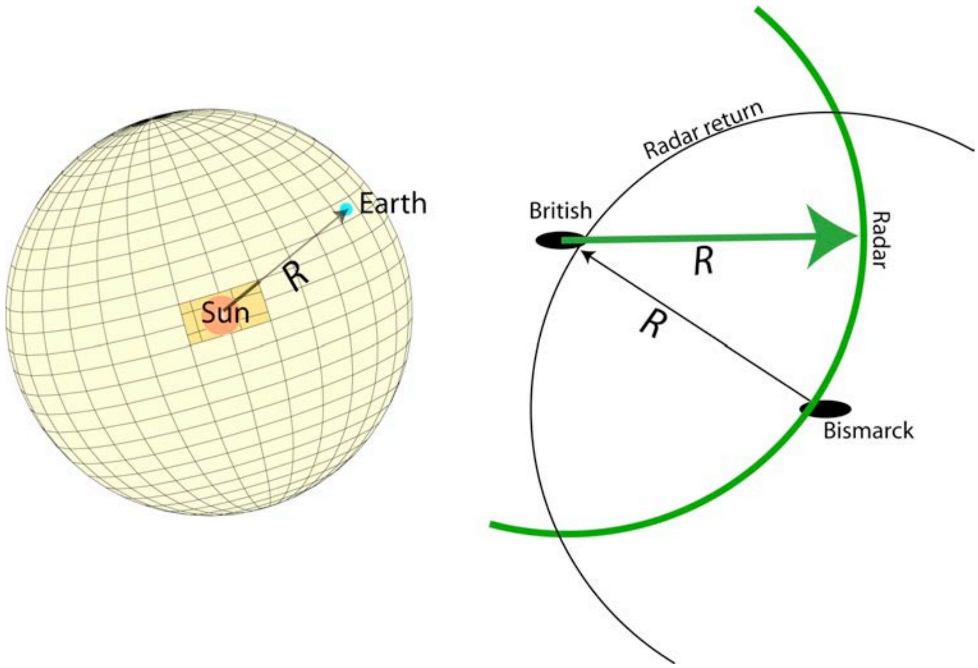


Figure 17: The Sun illuminates an imaginary sphere of radius *R* containing the Earth (left). Map view of a British radar ship and the Bismarck. The Bismarck detected the strong signal of the radar. The British ship could not detect the weak reflection.

These facts were not common knowledge even in World War II. The German battleship *Bismarck* was sunk because its captain was ignorant that they apply to radar (Figure 17). The *Bismarck* was still detecting British radar and did not keep radio silence because the captain assumed they were being watched anyway. The ability of the *Bismarck* to detect British radar decreased with distance squared, but the ability of the British to detect the *Bismarck* decreased with distance to the fourth power.

Returning to stars and planets, the light from an object of constant disk angle decreases inversely with the radius squared. Equivalently, to provide constant brightness as observed on the Earth, the disk angle of the stars must increase with radius squared. Galileo pointed out that the brighter stars are much further away than Jupiter but about as bright. For the stars to shine by reflected light from the Sun, their disks would have to be huge; instead they are too narrow to resolve even with a telescope. He then pointed out that a telescope is unnecessary. One can constrain the disk sizes of stars with the naked eye (Figure 18).

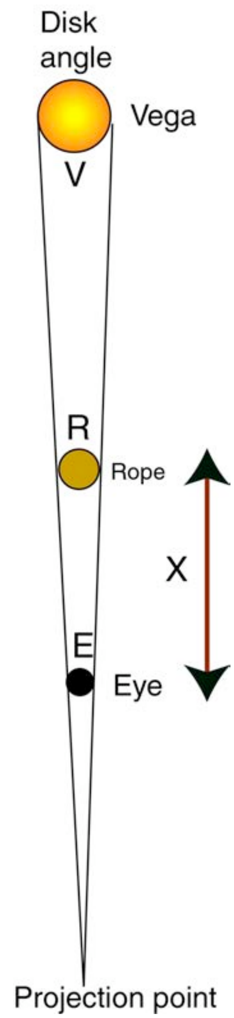


Figure 18: The disk of the star Vega is just visible on both sides of the rope of diameter R at is distance X from the eye of diameter E. With some geometry give in the activities, one obtains the disk angle of the star V. The measurement fails in practice because the disk angles of stars are too small, but one does obtain an upper limit on the disk angle.

Galileo tried using a rope to do this. He walked away until he could see the star on both sides of the rope and computed angles accounting for the finite width of the pupil in his eye. He got 5 arc second for the bright star Vega. This result seems reasonable for an actual experiment. If he used a 1-centimeter diameter rope (with a little calculation for a quick example), the focal point needs to be 400 meters from the rope and the observer about half as far. You can do it yourself with a planet. Galileo did not dare to push the argument any further, leaving the question of whether the Sun is a star unmentioned, but within the grasp of alert readers.

Despite the approval of the censors, Galileo had gone too far for the Church. His just estranged friend, the Pope, felt betrayed by the book and humiliated by Simplicio's final speech that man cannot fully comprehend God's infinite power. It was obvious to all that Galileo's book advocated Copernicanism in Italian, the language of the streets. Galileo was called before the inquisition in Rome. He knew from Bruno's fate that these people were serious. It was the middle of the Thirty Years War (1618-1648). The Church's position in central Europe was dire. It could not afford to look weak once it had expressed dogma on the Copernican system.

Practical men held the day and reached a plea bargain. The Church avoided having to burn alive an internationally known scientist for all of Europe to see. Galileo knew he had mortally wounded geocentric astronomy and that the Church would not be able to put his message back in the bottle. By the time his book was banned, there were few unsold copies to seize. Galileo's life ended in house arrest where he founded mechanics and material science.

The chill spread far and wide from Galileo's villa. René Descartes (1596-1650), a firm Copernican who was sure the Sun is a star, deferred publishing his magnum opus on *The World*. He eventually sought intellectual safety in Sweden. Like Copernicus before him, he perished and then published. Implicit is that they did not fear the powers of Churchmen in the hereafter but did fear them in the here and now. The *Dialogue* remained banned until 1835, long after all Catholic astronomers accepted that the Sun is a star and the Earth a planet. Galileo was for the most part rehabilitated in the twentieth century.

Retrospective and lunar occultations. Modern astronomers have studied Vega. It is 1.6 million times as far away as the Sun or 25 light years. (From now on, I use the astronomical convention of calling the distance between the Earth and the Sun, an astronomical unit, abbreviated as AU.) Vega's diameter is about twice that of the Sun. Its disk angle is thus the Sun's disk angle one-half degree or 1800 arc seconds times 2 divided by the distance ratio 1,600,000. This yields 0.002 arc second so that Galileo was off by a factor of 2500. His 1-cm diameter rope would have to have been 1000 km away to work.

Galileo's method failed because he did not understand that bending (refraction) of light in the atmosphere causes stars to twinkle. That is, subtle variations of temperature in the atmosphere act like moving lenses. The effect is like desert mirages, only weaker; it is similar to a point at the bottom of a wavy swimming pool appearing to move around. The path that the light takes through the air moves around (by less than a meter) too fast for the eye to resolve. The point of light comes in from slightly different directions at each "instant". The disks of planets are larger than the range of incoming directions so they do not twinkle. Viewed with modern photo-electronics, the instant image of a star is effectively a point. Modern telescopes combine the point of a "guide" star to align numerous brief images, using adaptive optics to continually adjust the mirrors. If no guide star is in the view field, they may use lasers to create an artificial point source 90 km above the surface.

Galileo could have done much better using lunar occultations than he did with his rope. There is no documentation on this matter so he most likely did not think of it. Had he thought of it, he could have easily passed the matter off as highly technical astronomy and left the inferences to the reader. If the Church had tried to suppress the idea, it would now be a *cause célèbre*. It was well known by the time of the ancient Egyptians that stars disappear behind the disk of the Moon. About an hour later, the star emerges on the other side of the Moon (unharmed to the relief of the ignorant). Watching an occultation sparked Copernicus' interest in astronomy.

It is easy to constrain disk diameter of stars with occultations (Figure 19). The Moon moves across the sky with a known rate. It circles the sky, 360 degrees each with 60 arc minutes and each with 60 arc seconds each month (crudely 30 days times 24 hours with 60 time minutes and 60 time seconds). That is, (360*60*60)/(30*24*60*60) or 0.5 arc second per time second. A fixed star blinks off in our reaction time of about 0.1 time second so its disk diameter is less than 0.05 arc second. Good timing is not essential to get this quick estimate and the gist of its implications.

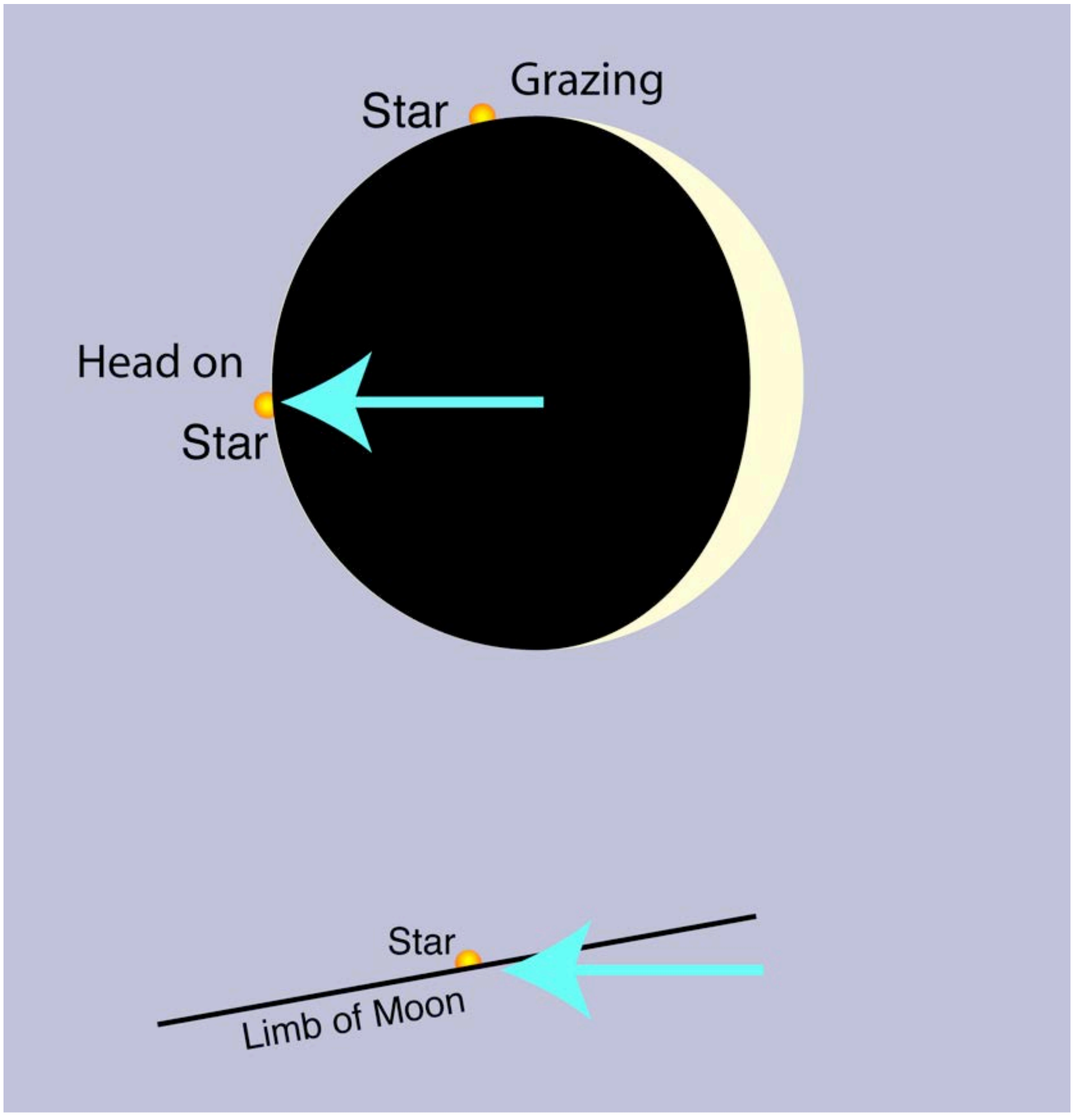


Figure 18: You can watch the occultation of a star by the Moon. The waxing Moon moves essential in the direction that the horns of the crescent point. The star blinks off quickly when it is covered by the limb of the Moon in a head-on occultation. This allows one to constrain the disk diameter. The star dims more slowly when its disk appears to approach the Moon at a near-grazing angle. The dimming time then can be estimated with the naked eye for certain giant stars.

The disk diameter of the Sun is 1800 arc second. If we assume that the occulted star has the same diameter as the Sun, it is at least 1800 divided by 0.05 times as far away, or 36000 AU. The parallax angle is smaller than 1/36000 radians [small angle formula in Primer on Geometry] or 6 arc seconds which is way too small to be resolved in 1629.

This suffices to silence lack of annual naked-eye parallax as an objection to the Copernican system.

The occultations argument provides an underestimate of the actual distances to stars. The nearest star system is Alpha Centauri. It is 277000 AU (4.36 light year) away. We have underestimated the distance to the nearest stars by only a factor of eight, which is petty good considering we needed no equipment. Alpha Centauri is a triple star system with one member a little brighter than the Sun, one somewhat dimmer, and one too dim to worry about. It resides in the southern sky too far away from the Zodiac to get occulted by the Moon. It would appear to blink off in two steps if it were occulted. This is a standard way to detect double stars. It will cross the Zodiac in about 30,000 years.

I reverse this situation to an observer of a lunar occultation on a planet in the Alpha Centauri system. Our Sun's disk angle is 1800/277000 = 0.0065 arc second. The occultation time would be 0.0065/0.5 = 0.013 time second is easily measurable with modern equipment but not by the eye. We could marginally revolve this angle in a grazing occultation with the naked eye.

In contrast to stars, planets dim slowly over tens of time seconds. For example, the disk of Jupiter is about 40 arc seconds so it dims out in 80 time seconds. This effect was not noted in antiquity even though planetary occultations were observed. For example, Aristotle observed an occultation of Mars. Alert observation would have given the disk sizes of the outer planets and the variation of the disk size of Mars as it moves closer and further from the Earth. The crescent of Venus, however, is not obvious from occultations with the naked eye.

It turns out that naked-eye astronomers would have got an actual disk diameter estimate from observing occultations of the bright stars Aldebaran and Antares. These stars are red giants that are more than an AU wide and far more luminous than the Sun. Their disks are 0.03 arc seconds. This is too small to resolve in a head-on occultation, but can be resolved in a near-grazing occultation where they take about a second to dim. If one assumed incorrectly that they are nearby sunlike stars, one would underestimate the distance of the nearest stars by a factor of 5.

Serious mostly amateur astronomers provide valuable data on occultations. The International Occultation Timing Association coordinates efforts. Their website gives information on upcoming occultaions, nice if you do not want to wait too long outside. http://www.lunar-occultations.com/iota

Brightness and star distance: Starlight, star bright. In his *Conversation*, Kepler stated that the Sun is infinitely brighter than the stars. Foscarini noted dangerously that the Sun would appear as bright as a star if viewed from far enough away. Our Sun viewed from the Alpha Centauri system would be (remembering that brightness decreases with the inverse of distance squared) would be a star $1/(277000)^2$ or one 10 billionth as bright as the Sun viewed from the Earth. Conversely, if we can measure the apparent brightness of a sunlike star, we can estimate its distance to the Sun.

In the 1690s, the Dutch physicist Christiaan Huygens (1625-1695) made a serious try at determining the distance to nearby stars. He was a staunch proponent of many inhabited worlds. By then, it was a given that the Sun is a star. He viewed the Sun though tiny pinholes. He varied the disk angle of the pinhole until it was a bright as he remembered the stars were at night. (Don't try this unless you have a liking for guide dogs. Huygens defocused the sunlight with a lens or dimmed it with smoked glass in addition to using the hole.) The disk angle of the pinhole gave the disk angle of the star. He then did the same calculations that I discussed for occultations to get the distance to the star in AU. This presumed that luminosity per surface area of the Sun is the same as that of a star.

The method left much to be desired. We have already seen that the disk angles of stars are tiny and it is impractical to get a real pinhole far enough away. One also needs to remember how bright the star was at night. Huygens got about 28000 AU, which is too small by a factor of 10 for the nearest star.

The Scotsman James Gregory (1638-75) and Isaac Newton (1642-1727) used a planet for an intermediary, which can be seen at night along with the stars. Saturn is convenient as it is about as bright as the brighter stars and as it is far enough from the Sun that the distance difference between Earth to Saturn and the Sun to Saturn can be ignored in a quick calculation. The disk angle of Saturn is 17 arc seconds (8 times 10^{-5} radians) and is 9.5 AU away. It reflects about 1 billionth of the light emitted by a hemisphere of the Sun. (This suffices to show Bruno was right that stars are much brighter than planets.) The Sun would be this bright if it were the square root of a billion times further away than Saturn or 300,000 AU, a nice guess for Alpha Centauri. Both Gregory and Newton used Sirius, which unlike Alpha Centauri can be seen from the British Isles. Using more care than I did in my quick calculation, they got 83,000 and 1 million AU, respectively. They did not know that Sirius is a lot more luminous than the Sun. Its actual distance is 540,000 AU.

A candle makes a nice intermediary that does not require a telescope as noted by

Huygens. [see exercises at end of chapter]

How big are the other planets and the Sun?

One of the goals of astrobiology is to find planets similar in size to the Earth. Classical astronomers started the effort by attempting to get the distance to the Sun. They had diameter of the Earth and the distance of the Earth to the Moon in common units, like miles. The relative distances of the planets from the Sun, that is the distances in AU were known. They did not have a good constraint on what the AU is in miles. Their estimate was low by a factor of 15. Kepler attempted to improve on the classical result; he was still low by a factor of 10.

This estimate of the dimensions of the solar system must have had a strong influence on thinking once telescopes provided the disk diameters of the planets. The small (Kepler) value of the AU made the diameters (in miles) of the planets too small by a factor of 10. Mercury, Venus, and Mars were all tiny objects smaller than the Moon. The Earth was the same size as Jupiter and Saturn. Kepler gave the Earth a special place in his *Conversation*, as did Foscarini with the Sun.

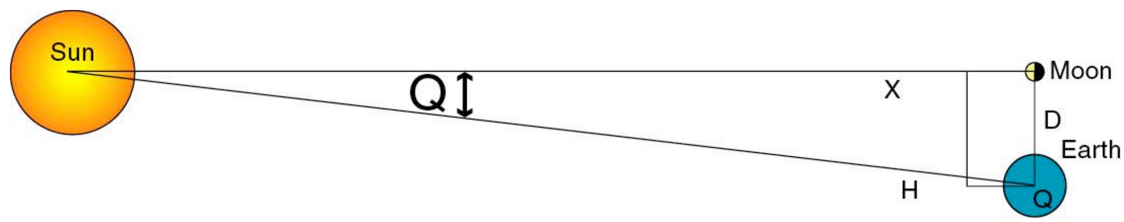


Figure 20: Quadrature of the Moon gives the Sun-Earth distance H (the AU) in terms of the Earth-Moon distance D. The angle at the Moon is a right angle. The angle at the Earth between the Earth-Moon line D and the Earth-Sun line H is slightly less than a right angle by the quadrature angle Q. Measurements in antiquity did show that the Sun was quite far away, but the rough surface of the Moon makes the method impractical for a precise measurement.

The Greek method is ingenious but does not work in practice (Figure 20). When the Moon is exactly half full, the lines from the Sun to the Moon and the Moon to the observed on the Earth form a right angle. The angle that the observer on the Earth sees is slightly less than a right angle. As the angles of a triangle add up to 90°, the difference of that angle from a right angle is the quadrature angle *Q*. One then can use similar triangles to compute the distance to the Sun in Earth-Moon distances.

The actual angle is about 0.2°, too small to be resolved with the naked eye. The Greek astronomer Aristarchus got 3° and Kepler got 2°. They should not have incorrectly measured the Moon-Sun angle by this amount. They missed the actual quadrature time by hours so quick reaction was not the problem. Impatience may have been. The waxing half-moon is in the sky with the Sun between about noon and sunset. One sees the actual waxing quadrature only (on average) once every four months and the waning one once every four months. The uneven surface of the Moon as seen by Galileo was a serious problem that persisted even with good telescopes. By the late 1600s, it was evident that the quadrature angle was small and could not be resolved.

A do-it-yourself project is to measure the quadrature time by naked eye or with binoculars. The quarter moon times in almanacs are the quadrature time where the Moon is directly overhead. Computer programs to obtain the time at another point are not readily available. Check only the day when a half-moon is to occur so you do bias your results.

It became evident that daily and geographic parallax must be used to determine the AU. Mars and Venus are the nearest planets, but they move across the background of fixed stars. The small variations from parallax were hard to separate from these motions. The astronomers used Mars where its motion changed from retrograde to prograde (or vice versa) when these motions are as small as possible and transits of Venus across the disk of the Sun. Martian data were easy to get but were considered unreliable. Transits of Venus occur in pairs separated by over 100 years.

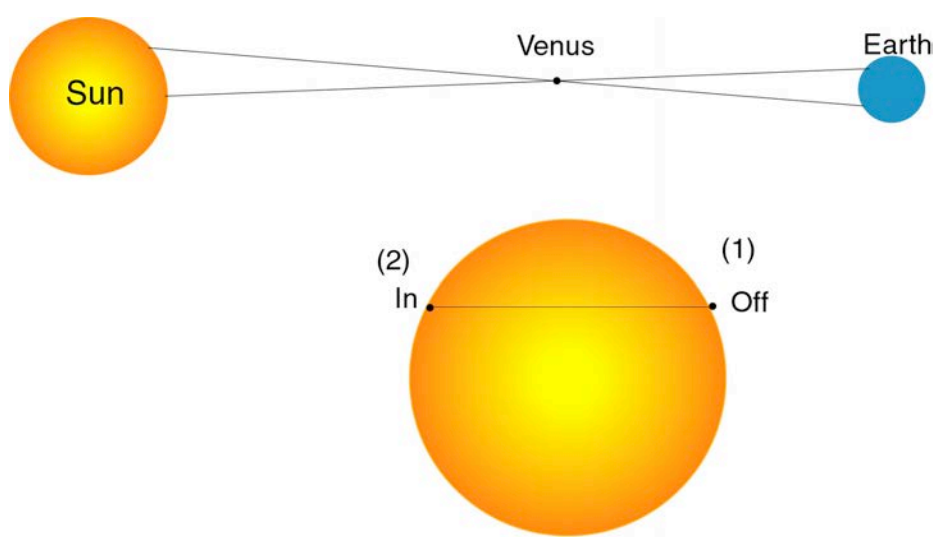


Figure 20: The Earth is a baseline for observing parallax during a transit of Venus. It is easier to record the time for Venus to cross the disk of the Sun than its precise positions on the disk. Two pairs of times are relatively easy to observe: (1) The limb of Venus is just fully off the disk of the Sun. (2) Venus is just fully on the disk.

In 1716, Edmond Halley presented a proposal to study the next transit of Venus. Like the return of his comet, it would not occur in his lifetime. The proposal had a distinctly modern bent. Here it is with my annotations in square brackets. This translation of Halley's paper is taken from the *Abridged Transactions of the Royal Society*, Volume VI, pp.243-249, published in 1809. The full text is at

http://sunearth.gsfc.nasa.gov/eclipse/transit/HalleyParallax.html

Here are NASA home pages for the 2004 and 2012 transits

http://sunearth.gsfc.nasa.gov/eclipse/OH/transit04.html

http://sunearth.gsfc.nasa.gov/eclipse/transit/venus0412.html

[Statement of Problem.]

It is well known that this distance of the Sun from the Earth, is supposed different by different astronomers. Ptolemy and his followers, as also Copernicus and Tycho Brahe, have computed it at 1200 semi-diameters of the Earth, and Kepler at almost 3500; Riccioli doubles this last distance, and Hevelius makes it only half as much.

[New Data that warrant further effort]

But at length it was found, on observing by the telescope, Venus and Mercury on the Sun's disk, divested of their borrowed light, that the apparent diameters of the planets were much less than hitherto they had been supposed to be; and in particular, that Venus's semi-diameter, seen from the sun, only subtends the fourth part of a minute, or 15 seconds; and that Mercury's semi-diameter, at his mean distance from the sun, is seen under an angle of 10 seconds only, and Saturn's semi-diameter under the same angle; and that the semi-diameter of Jupiter, the largest of all the planets, subtends no more than the third part of a minute at the sun.

[Halley must figure out how precise the proposed measurements need to be. This is the starting point of any modern proposal to collect improved data. He extrapolates from other data. He guesses using reasonable assumptions. His first assumption is that disk angle of planets as viewed from the Sun should be similar. If this were true the planets when viewed from a distant star would be equally bright.]

Whence, by analogy, some modern astronomers conclude that the earth's semi-diameter, seen from the sun, subtends a mean angle, between the greater of Jupiter and the less of Saturn and Mercury, and equal to that of Venus, viz. one of 15 seconds; and consequently, that the distance of the Sun from the Earth is almost 14,000 semi-diameters of the latter. [Modern answer ~23,400]

[Assumption: Planets are bigger than moons]

Another consideration has made these authors enlarge this distance a little more: for since the moon's diameter is rather more than a quarter of the earth's diameter, if the sun's parallax be supposed 15 seconds, the body of the moon would be larger than that of Mercury, viz. a secondary planet larger than a primary one, which seems repugnant to the regular proportion and symmetry of the mundane system.

[Assumption: Planets with moons are larger than those without moons.]

On the contrary, it seems hardly consistent with the same proportion, that Venus, an inferior planet [inferior here means nearer to the Sun than the Earth, Halley extends this to the common meaning of the word; this is an example where terminology affects thinking], and without any satellite, should be larger than our

Earth, a superior planet, and accompanied with so remarkable a satellite. Therefore, at a mean, supposing the earth's semi-diameter, seen from the sun, or which is the same thing, the sun's horizontal parallax, to be 12 seconds and a half, the moon will be less than Mercury, and the Earth larger than Venus, and the Sun's distance from the earth come out nearly 16500 semi-diameters of the Earth.

[Certain methods do not work reliably. We must try something else.]

I shall admit of this distance at present, till its precise quantity be made to appear more certain by the trial I propose; not regarding the authority of such as set the Sun at an immensely greater distance, relying on the observations of a vibrating pendulum, which do not seem accurate enough to determine such minute angles [Used for timing the motion of Mars in the sky and hence the diurnal/geographic parallax of Mars. They measured the times that Mars and stars in its field of view crossed a meridian (longitude line) in the sky.]*;* at least, such as use this method will find the parallax sometimes none at all, and sometimes even negative; that is, the distance will become either infinite, or more than infinite, which is absurd. [It turned out that the better analyses of Mars gave about the right answer, but Halley did not know this. Bad data can taint the good.] And it is scarcely possible for any one certainly to determine, by means of instruments, however nice, single seconds [of arc], or even 10 seconds; and therefore, it is not at all surprising, that the exceeding minuteness of such angles has hitherto baffled the many and ingenious attempts of artists.

[Discussion of use of transit of Venus follows, one paragraph later.]

There remains therefore Venus's transit over the sun's disk, whose parallax, being almost 4 times greater than that of the sun, will cause very sensible differences between the times in which Venus shall seem to pass over the sun's disk in different parts of our Earth.

[Needed instruments and techniques]

From these differences, duly observed, the sun's parallax may be determined, even to a small part of a second of time; and that without any other instruments than telescopes and good common clocks [precise clocks that could keep time on a long sea voyage did not yet exist], and without any other qualifications in the observer than fidelity and diligence, with a little skill in astronomy.

For we need not be scrupulous in finding the latitude of the place, or in accurately determining the hours with respect to the meridian [longitude]; it is sufficient, if the times be reckoned by clocks, truly corrected according to the revolutions of the heavens [local day time], from the total ingress of Venus on the sun's disk, to the beginning of her egress from it, when her opaque globe begins to touch the bright limb of the sun; which times, as I found by experience, may be observed even to a second [See Figure 20].

[The geometric details follow.]

French and British scientists observed the transits of 1761 and 1769. They did this when their countries were at war. When the observers finally returned with tales of the far reaches of the Earth, the data yielded essentially the present result that the AU is 150 million kilometers in modern units.

Perhaps ironically, Bruno may have guessed the right result when he visited Oxford.

The English Bishop Francis Godwin (1562-1633) wrote the adventure book "The Man in the Moone." which was published in 1638 after his death in 1633. Godwin was a student at Oxford during Bruno's visit, but it unclear what he got from Bruno. His adventurer contemplates the difficulty of the fixed stars circling the Earth each day when he is part way to the Moon:

> "those same huge bodies of the fixed stars in the highest orbe, whereof divers are by themselves confessed to be more then one hundreth [hundred] times as bigge as the whole earth" (Modern number for the Sun is 109) (page 70)

Link to text of book:

http://e3.uci.edu/clients/bjbecker/ExploringtheCosmos/week2e.html

Bottom line, update, and parallax

Actual measurements of parallax proved difficult to make. By the late 1600s equipment was good enough to try. In 1725, James Bradley conducted a careful experiment. He placed a fixed telescope so that a bright star would pass vertically overhead and measured its position each time it passed the north-south meridian. (Bright stars are in fact visible with telescopes during the day.) He found out that the star (and all others subsequently looked at) swept out an ellipse in the sky of about 40 arc seconds. This was too large for the expected parallax and in the wrong orientation. He had discovered stellar aberration (Figure 12). Light travels at a finite rate, which was already known in 1725 from observing the moons of Jupiter. The effect is much like rain hitting a car. If it is falling straight down when the car is stopped, it will appear to fall toward the windshield when the car is moving forward and into the back windshield when it is backing. This was conclusive, though belated, evidence that the Earth in fact circles the Sun. Bradley obtained an accurate estimate of the speed of light from the effect.

It was thus necessary to use the apparent motion of a nearby star relative to distant background stars to measure parallax. This was not done until 1837. The main problems were that one needs a telescope that does not distort when it magnifies and that the other stars are moving with respect to the Sun. Halley discovered this effect called proper motion by comparing star charts from antiquity with modern star positions. Nearby stars typically cover several AU in a year, so their proper motions against the distant background in that time are more than their parallax. One needed to observe a star for several years to resolve parallax.

There are good measurements of the diameters of sunlike stars out to 30 light years. This work confirms that the Sun's luminosity to diameter ratio is that of similar stars. The sophisticated equipment called an interferometer takes advantage that light behaves as a wave for those with some physics.

Astronomers now have a good picture of the vastness of space and continue to augment the accuracy of their methods. They use parallax to obtain the distances of nearby stars. They obtain objects of known brightness (standard candles) from this data and work out. The method is basically that used by Newton and Huygens, except we now know that assuming all stars are like the Sun makes a poor standard candle.

We live in the outer part of the disk of a spiral galaxy with hundreds of billions of stars. There are hundreds of billions of other galaxies out to distances of over 12 billion light years. The universe is vast. Our technology limits us to looking for life nearby, around the neighboring stars seen by Galileo and Halley.

Primer on Seasons

The primer provides an overview for the discussion of procession of equinoxes in the following Chapter 3. In addition, the seasonality of climate affects the habitability of a planet. The primer introduces the basic terminology and geometry.

The Copernican system explains seasons from the inclination of the Earth's axis to the plane of its orbit around the Sun (Figure S1). To the degree that one can observe with the naked eye over one's lifetime, the positions of the orbital plane, the plane of the equator, and the celestial pole do not change. The relative orientation of the sun and axis do change during the year.

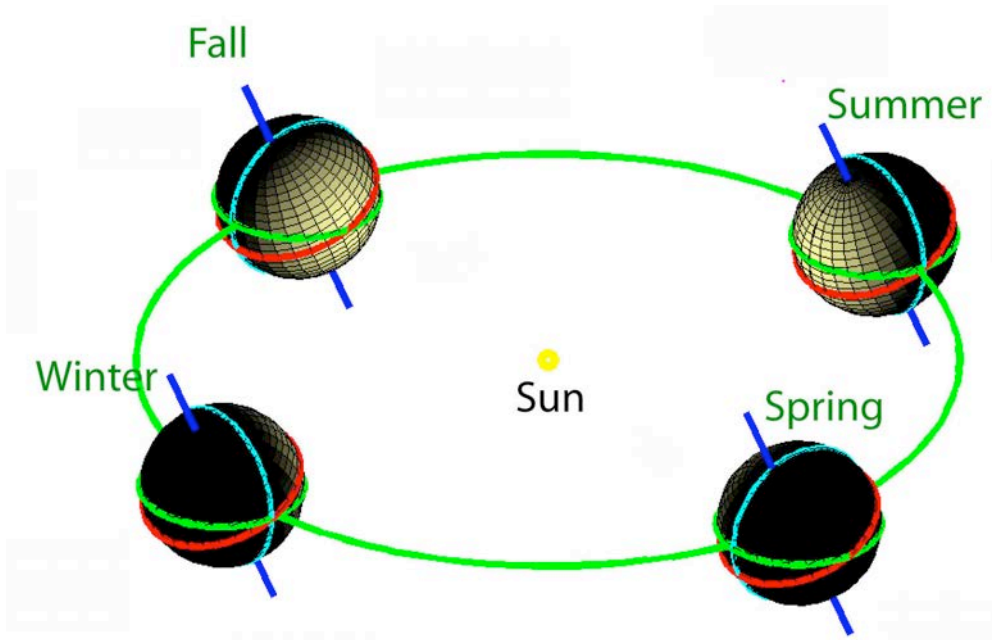


Figure S1: The orbit of the Earth around the Sun viewed from space from a place north and outside its orbit. The Earth rotates counterclockwise about its axis that stays aligned with a point in the sky. This produces day and night. It resolves counterclockwise through its orbit, producing the seasons. The equator is shown in red and the projection of the plane of the Earth's orbit, the ecliptic, through the Earth's center in green. The orbit is nearly circular and the Earth is slightly closer to the Sun in the northern winter than in the summer. Note that a circle looks oval when view obliquely; see Primer on ellipses in Chapter 3 for more. The Earth would be a speck if drawn to true scale on its orbit.

Starting at the summer solstice around June 21, the Sun is at its maximum position north of the equator as noted above (Figure S2). It is directly overhead at the Tropic of Cancer, 23.5°N. The region north of the Arctic Circle (90-23.5 = 66.5°N) receives 24 hours of sunlight. The longest daily period of daylight occurs on that day between the Tropic of Cancer and the Arctic Circle. Most of the Sun's rays fall within the northern hemisphere bringing warmth. The southern hemisphere receives less light and is cold. The region south of the Antarctic Circle (66.5°S) experiences 24 hours of darkness. The Sun is in Gemini at the northern summer solstice.

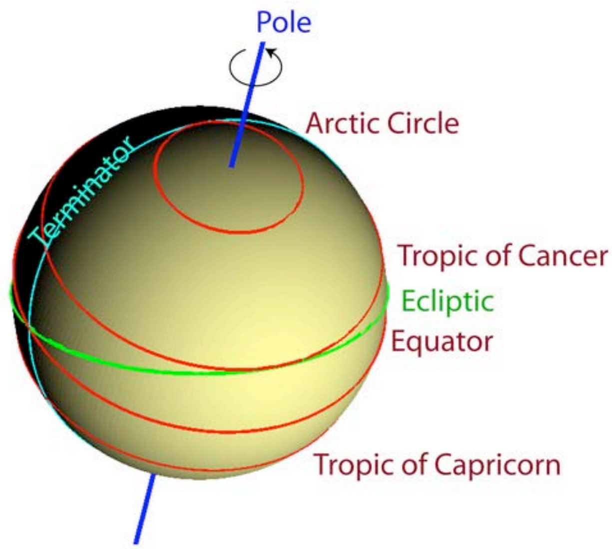


Figure S2: The lit side of the Earth viewed from space at the northern summer solstice. The terminator separates the lit and dark sides. The ecliptic is the plane of the Earth's orbit through its center. There is 24 hours of daylight north of the Arctic Circle. The Sun is directly overhead at noon at the place where the Tropic of Cancer intersects the Ecliptic. The Earth viewed from the Sun moves to the left in the direction of the intersection of the terminator and the Ecliptic.

At the fall equinox, the Sun is in the plane of the Earth's equator. It is directly overhead there at noon. It moves along the horizon at both poles. Everywhere on the Earth gets 12 hours day and 12 hours night. The Sun is in Virgo.

At the northern winter solstice, the Sun is as far south as it gets in the sky. The southern hemisphere gets most of the Sun's rays. The Sun is in Sagittarius. The Sun is again directly over the equator at the spring equinox (Figure S3). The Sun is in Pisces.

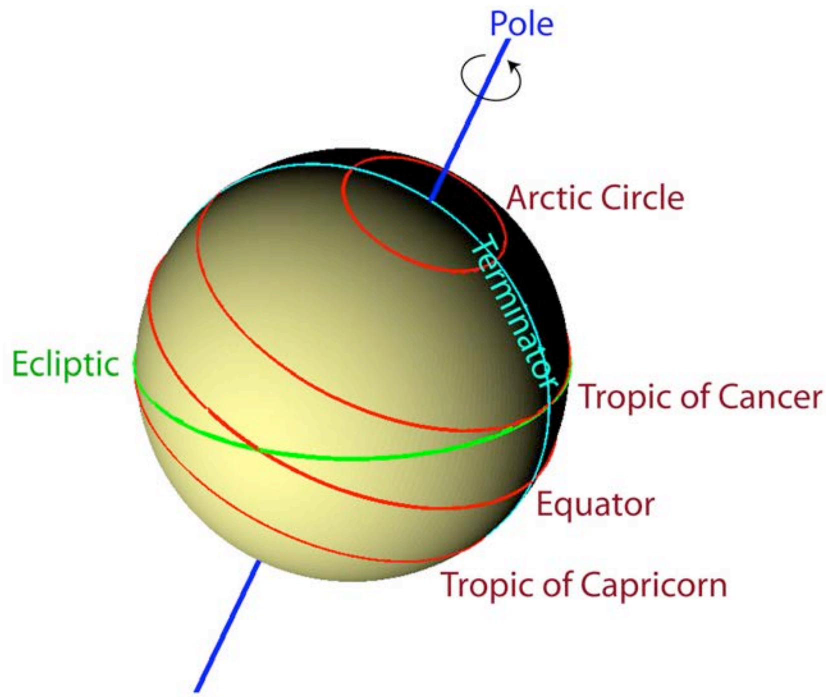
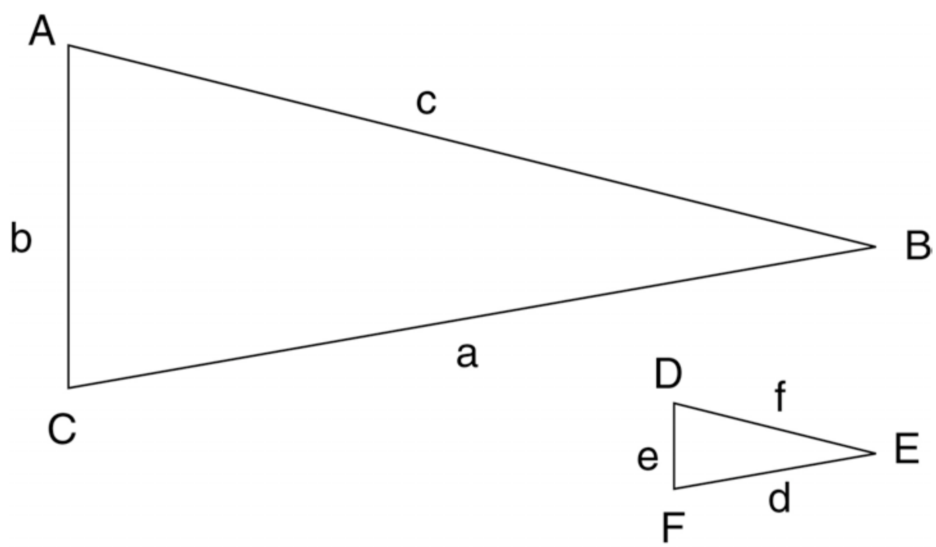


Figure S3: The lit side of the Earth viewed from space at the northern spring equinox. There are 12 hours of daylight everywhere. The Sun is directly overhead at noon at the place where the Equator intersects the Ecliptic. The Earth viewed from the Sun moves to the left in the direction of the intersection of the terminator and the ecliptic, which is hidden from view on the backside.

You can make a crude sundial and track the motion of the Sun through the sky during the day and the year with a stick driven vertically the ground, like a phone pole. At dawn, the Sun is in the east and the shadow points west. At noon (if you are north of the Tropic of Cancer), the Sun is in the south and at its high point in the sky. The shadow points north and is at its shortest point of the day. You can find north by drawing concentric circles around the stick. The noon shadow is longest at the winter solstice and shortest at the summer solstice. As already noted, the direction of the rising Sun varies slowly throughout the year. It is north of east in the spring and summer, due east at the equinoxes, and south of east in the fall and winter. Sunset is north of west in the spring and summer and south of west in the fall and winter.

Primer on Geometry

In case, you did not take or are rusty on geometry, the basics are quite simple. Complications come in actually measuring angles. That is, modern surveying and astronomical equipment is very sophisticated. The nonmathematical reader can be content that this overview shows that things can be worked out.



Triangles. Much of classical geometry involved congruent and similar triangles. Congruent triangles are exactly the same and can be exactly overlain. Two triangles are congruent if all three sides are the same. A triangle with rigid sides cannot be deformed; engineers use this feature to keep structures strong. Triangles can be shown to be congruent if two sides (say a and b) and the included angle (C) are the same or two angles (say A and C) and the included side (b) are the same. Similar triangles have the same shape, but may be different sizes. Two triangles are similar if they have two angles that are the same. Here C and F and A and D are the same. This implies that B and E are the same because the angles add up to 180°. Similar triangles allow small objects to be used to measure large distances on the Earth and inaccessible distances in astronomy. For example, if side b represents the disk of the Sun, sides a and c are the Earth-Sun distance, called an astronomical unit (AU). The diameter of the Sun in astronomical units is (side b)/(side a) which is also (side e)/(side d). We can construct the triangle DFE so that the angle E is the same as the disk angle of the Sun, 0.5°.

Radians and small angles. In astronomy, one of the angles is frequently very small, like the disk of a star viewed from the Earth, while the other two angles are almost 90° right angles. Then it is convenient to use radians to measure angles. (This is the RAD setting on your hand calculator.) We define the angle so that the ratio of the small side to large side is the small angle. We can get the relationship of radians to degrees by noting that the circumference of a circle is 2 times pi times the radius, $2\pi R$. This gives that 1 radian is about 57 degrees or for astronomical purposes that 1 arc second is 2.778 x 10^{-4} radians or that 206300 arc seconds make a radian. Typically, the astronomer gets to measure the small angle and the short side. In the case of annual parallax, the short side is one AU and the angle is the parallax angle. To get the distance of the star in AU one divides 1 AU by the parallax angle in radians. Equivalently one may divide by the parallax angle in arc seconds and multiply by 206,300. Astronomers call the distance where the parallax angle is 1 arc second, a parsec, 206,300 AU.

Notes

A purely solar pagan calendar divided the year into 8 unequal intervals. The solstices and equinoxes provide 4 dates. The other 4 dates are when the length of day is midway between that of the solstice and the equinox. Christmas has displaced the winter solstice. Easter follows the spring equinox. The Ground Hog Day (February 2), May Day, the summer solstice, and Halloween retain pagan trappings, but have lost Christian theological connotation in much of the United States. Many Latinos observe the Day of the Dead (November 1) after Halloween. The fall equinox is associated with no widely observed modern Christian holiday, but defines the Jewish New Year. August 1 is Lammas, in England marking the start of the wheat harvest.

Galileo published his final work "Two New Sciences" with the Dutch Protestant publisher Louis Elsevier. Galileo safely pretended that his draft was published without his permission. Elsevier today is a major publishing house for scientific journals and books.

Do-it-yourself activities

Ancient navigation, timekeeping, and astronomy required very little equipment and the naked eye. It makes for many do-it-yourself activities.

Figure out which way is north. If you want to do this yourself, the Sun reaches its highest point in the sky (at northern temperate latitudes) when it is due south at astronomical noon. A simple way to do this is to drive a vertical stake into flat ground. Make a drafting compass with chalk on pavement or a pointed stick or dirt to draw circles around the stake. The shadow is shortest at noon. Marking the end of the shadow every few minutes will help you see when it is shortest. Use shadows so you do not have to look directly at the Sun. Measure the stick and the length of the shadow and plot them to scale on graph paper. You then can use a protractor to get the angle. A magnetic compass gives magnetic north. Topographic maps give local difference of magnetic and true north.

Finding your location: Sailors needed to find the difference between their location and the known location of a port. You and a friend who is some distance away can perform their task.

Record the time and the angle above horizontal when your star is due north or due south (at the meridian). You can measure angles with a plumb bob or carpenter's level to get vertical and horizontal and rulers to obtain the vertical and horizontal sides of the right triangle with the hypotenuse pointed at the star. You then can make a scale drawing and use a protractor to measure angles. Or you can site along protractor directly. If you have a surveyor's transit, you will be able to do as well as the 1700s sailors. If the star is in the south, the angle is recorded as if it was measured from the north. For example, a star 30° above the southern horizon is 150° above the northern horizon. Compare the results with your friend's. The latitude difference is the difference between your two angles. The larger angle is further north.

You require timing to measure differences in longitude. Synchronize your watches and record the time that your star crossed the meridian. The time difference gives the longitude difference with the later time to the west.

A star reappears at the meridian every 23 hours, 56 minutes, and 04.09074 s. (The Sun returns to the meridian in 24 hours because its position in the sky moves west during that time.) If you have recorded the time difference in minutes, the longitude in degrees difference is the time difference times 0.25068. (Time west - time east) in minutes x 0.25068 = longitude difference. (The factor is 0.25 if you use the Sun.) Now is a good time to use your GPS receivers or a map to see how well you did.

During the age of exploration, the sailors could measure latitude, which does not require timing. Timing was a serious problem that made it hard to measure longitude. Early maps are distorted because the longitudes are in error. One of Halley's day jobs was to get navigation right. For a fixed location, one could use the start of an eclipse of the Moon, which by 1700 could be precisely predicted. One could also use tables for the moons of Jupiter or more complex methods based on the phase of the Earth's Moon. None of these were particularly practical for sailors at sea. Accurate clocks were the solution. The sailors had tables of when stars crossed the meridian at Greenwich (near London) so no radio was necessary.

Getting the radius of the Earth. The ancients determined the radius of the Earth with a procedure analogous to that for determining latitude. They measured the angle of the Sun from the vertical at two locations along a north-south line. They recorded the distance between the stations and the difference between the angles. It then became a matter of using a similar triangle to get the radius and the circumference of the Earth. They did not do this very carefully but they still got about the right answer.

To try this yourself, you need to measure the distance between two places in a north-south line. Your car will do if you live where the roads run north-south. Then measure the angle of the Sun at noon or a star crossing the meridian at each end of your line as explained at the top of this box. Again wait until you are done to get your GPS out.

Galileo's method of measuring the disk angle of the star Vega (Figure 18). The disk angle of the star is also the angle from a projection point behind your eye through the eyeball and the rope. The distance between your eye and the rope is X, which you can measure.

The rope diameter is R. The diameter of your pupil is E. With a little algebra, the disk angle is (R-E)/X

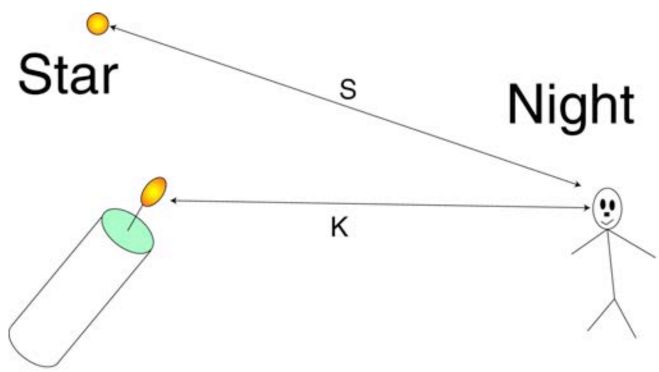
ANSWER = (Rope - Eye) / distance

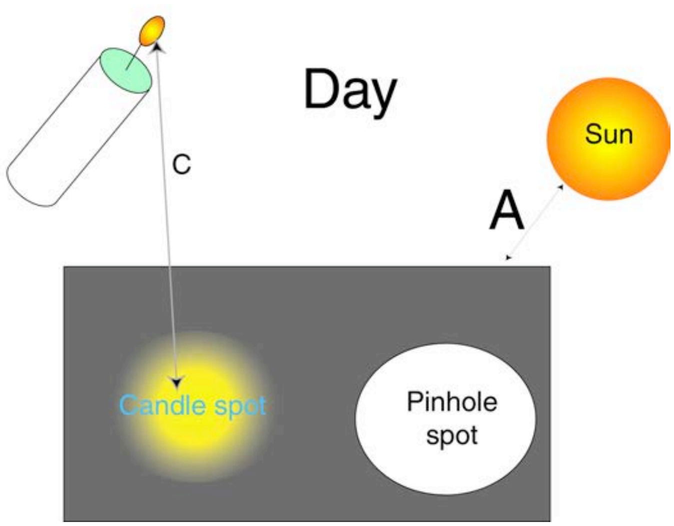
(________ - __________) / _________________

Put R, E, and X in common units. This gives the disk angle in radians. Multiply by 3438 to convert to arc minutes.

This can be tried with a planet, as the disks of stars are too small to actually measure in this way. Measure your pupil (have a helper) after have you have been in the dark for a while. A taut rope works but a moving one will not. So does a railing on a building if it is in the right place. A phone wire will work if you just want to see the effect. Walk back until you just see light on both side of the rope. You can use a right triangle to measure X if the rope is well off the ground and you want a more precise answer.

Estimating the distances to stars using equipment available to the Greeks. This is a safe version of Huygens' experiment. All you need is the concept that the Sun is a star. A candle and a pinhole camera are needed.





Night project. Find a place where you have a reasonable view of the sky. Wait until your eyes can see the brighter stars. Light the candle and have your assistant move away until the candle is a bright as the brighter stars and record this distance. Pacing it off should be OK. Use the meter stick to determine the length of your pace.

K = ________________________

Day project. Use a pinhole to make a dim spot of sunlight. Move the candle until it produces a spot a bright as the pinhole. The spot will not have a sharp edge. You will probably need to get the pinhole a few meters away from the spot to get it dim enough.

Record

diameter of pinhole H = ____________________

diameter of pinhole spot P = __________________

distance of candle from its spot C = ________________

The color of the candlelight is not the same as the sunlight, but try to estimate the brightness of the spot anyway. In fact, a pinhole is an effective tool for seeing the difference in color between sunlight and other light sources.

Calculation. The brightness of an object is proportional to its luminosity and inversely proportional to its distance squared. At night we have that

Brightness = (Luminosity of star)/S2 =(Luminosity of candle)/K2

During the day we have that the apparent distance D to the Sun is the spot pinhole diameter ratio D=P/H in AU. (The true distance is 1 AU or A in the Figure.) The brightness relationship is

Brightness = (Luminosity of Sun)/D2 =(Luminosity of candle)/C2

We assume that the star and the Sun are equally luminous. This is the simplest guess to make. Then (with a little math) the distance to the star in AU is

S = (K x P) ÷ (C x H)

S = (_________ x ________) ÷ (__________ x ___________) = __________

You now have the distance to the star in AUs. You can now compare it with Newton's Bradley's, and Huygens' estimates. You did not need a telescope. In fact, this experiment could have been done in antiquity if anyone thought of it.

Scientists typically try to see the sources of uncertainty in their experiments. As you used your naked eye, it is hard to be quantitative, but your experience is relevant. Which part of your measurement do you think was least accurate?

Brightness of sunlight on the other planets. The pinhole is an effective means of controlled dimming of sunlight. Mars, Jupiter, Uranus, Neptune are 1.5, 5.2, 9.6, 19, 30 AU away from the Sun, respectively. Move the pinhole away from the paper so that the ratio of the spot diameter to the hole diameter is 1.5. The spot is then as bright as sunlight on Mars. Repeat this for the other planets. Is there enough light to read by on a moon of Neptune?

Venus is 0.7 AU from the Sun and Mercury is 0.3 AU. Measure the diameter of a reading glass. Project the light on the paper so that the spot diameter is 0.7 times the lens diameter. This is how bright sunlight is at the distance of Venus. Repeat with the spot focused to 0.3 times the lens diameter to show the brightness of sunlight on Mercury.

Computer graphics exercise. The apparent paths of the planets through the sky may for good exercises for the reader versed in computer graphics and geometry. A start is to represent Mars and the Earth with circular but slightly inclined orbits. That is, the orbits are on different planes. Have the planets move around their orbits at constant rates. You can plot the loop traced by Mars in the sky during its retrograde motion.

Geometry exercise. The gnomon (the object that casts the shadow) of a modern sundial is parallel to the Earth's axis. There is often a screw to adjust it. The reader familiar the geometry can quickly show why this alignment is advantageous.